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EFFICIENCY OF BENT UP-BARS AS SHEAR REINFORCEMENT

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EFFICIENCY OF BENT-UP BARS AS SHEAR REINFORCEMENT

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1. INTRODUCTION

During the past seventy years more than 1000 shear tests have been carried out on reinforced concrete beams, and a correspondingly large number of articles has been published on the subject. Despite this there have been very few systematic investigations into the relationship between the shear strength and the type and degree of the web reinforcement. According to a report by ACI-ASCE Committee 426 [1] (formerly Committee 326): »No recent test data are available for beams with bent bars or for beams with combined bent bars and stirrups».

In the following, the effect of shear reinforcement in the form of bent-up bars on the shear strength will be discussed, partly on the basis of tests published since 1962.

In sections near the point of contraflexion of a reinforced concrete beam, the longitudinal reinforcement becomes partially superfluous as the shear force increases. This means that a saving can be achieved by bending-up some of the longitudinal bars and using these as shear reinforcement. This method is particularly suitable in the case of continuous beams, in which the negative moments at the intermediate supports necessitate reinforcement at the top of the beam.

However, certain disadvantages attach to the use of bent-up bars as shear reinforcement. For instance, as shown by tests carried out by Leonhardt and Walther [2], inter alios, considerable compressive stresses occur in the concrete at the bending points of the bars. Especially when there is a small width of web, these stresses can result

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in wider cracks and a tendency for the concrete to split in the plane of the bent-up bars. The above tests show that this type of failure can be partly prevented by providing local transverse reinforcement in the form of stirrups.

When the longitudinal reinforcement is bent up and used as shear reinforcement, the bending-up must first be done at the point along the bars at which they are no longer needed as bending reinforcement. It is therefore necessary to take into account the increment in the tensile force in the reinforcement resulting from the diagonal cracking. These factors have been treated in detail by Leonhardt [3], inter alios. If the longitudinal bars are bent up too early, the ultimate strength of the beam will be considerably reduced, as has been shown by tests carried out by Ferguson and Matloob [4], Leonhardt and Walther [5], et al.

2. *THEORY*

When the bent-up reinforcement has been correctly arranged, as described above, its contribution to the shear strength is normally calculated by means of the equilibrium equations for the diagonally cracked section.

Assuming yielding in the reinforcement (see fig. 1) the contribution ΔV_b of the individual bar to the shear strength is:

$$\Delta V_b = A_b f_{yb} \sin \alpha \tag{1}$$

where

A_b: cross-sectional area of the bar,

f_{yb}: yield stress in the bar,

α: angle between the bar and the axis of the beam.

For uniformly distributed bent-up bars with the same yield stress f_{yb} and angle α, summation over the diagonal plane (see fig. 1) gives the total contribution V_b of the bent-up bars to the shear strength as

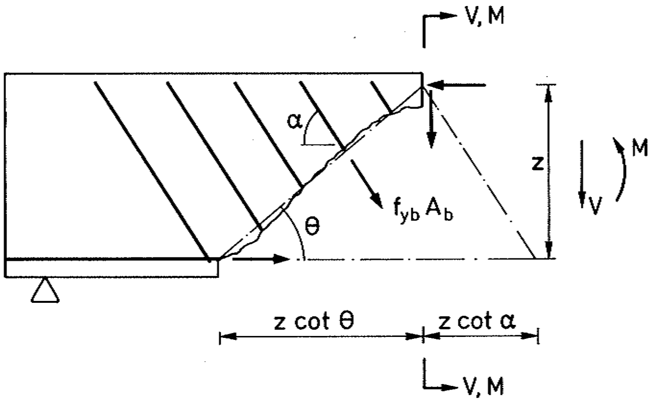

Fig. 1. Equilibrium conditions at an inclined plan.

$$V_b = \frac{A_b f_{yb}}{s} z \sin\alpha (\cot\alpha + \cot\theta) \qquad (2)$$

where

- s: distance between the bent-up bars measured along the axis of the beam,
- z: the lever arm,
- θ: idealized angle between the diagonal crack and the axis of the beam.

θ is normally put at 45°, which means that we get from (2)

$$V_b = \frac{A_b f_{yb}}{s} z (\sin\alpha + \cos\alpha) = K r_b f_{yb} b_w z \qquad (2a)$$

where

- K: $\sin\alpha (\cos\alpha + \sin\alpha)$,
- r_b: shear reinforcement ratio $= \dfrac{A_b}{s\, b_w \sin\alpha}$,
- b_w: width of web.

Apart from some discussion on the K-factor, equations (1) and (2a) have been used in the calculations of the contribution of both bent-up bars and stirrups to the shear strength ever since Ritter and Mörsch formulated the Truss Analogy.

The K-factor appears directly from the equilibrium equations, and its importance to the stirrup reinforcement was pointed out by Richart [6] as early as 1927. However, the K-factor is frequently neglected or overlooked, because it is equal to unity when $\alpha = 45°$ or $90°$, which are the most common cases.

Only K. W. Johansen [7], [8] and [9] seems to have challenged the above method of calculating the contribution of the bent-up bars to the shear strength, in that he states that yielding cannot occur in the bent-up bars.

The following considerations regarding the stress field at the point of bending-up will clarify this question (see also [10]).

For a section of a beam between two parallel diagonal cracks, as shown in fig. 2, the following expressions can be obtained by means of the equilibrium equations:

$$f_{b1} A_b \cos\alpha + f_c b_w x \sin\theta \cos\theta = f_{b2} A_b + \Delta N_x \qquad (3)$$

$$f_{b1} A_b \sin\alpha - f_c b_w x \sin^2\theta = 0 \qquad (4)$$

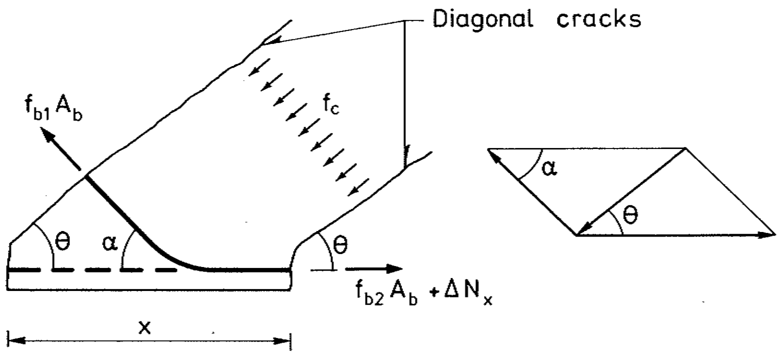

Fig. 2. Equilibrium conditions in the region of the bent-up bars.

where

f_{b1}: tensile stress in the bent-up bar at the diagonal crack,

f_{b2}: tensile stress in the bar just before the bending-up point,

f_c: compressive stress in concrete web, calculated to be uniformly distributed over the width of the web,

x: distance between the two diagonal cracks under consideration, measured along the axis of the beam,

ΔN_x: difference in the tensile force over the distance x for the straight part of the longitudinal reinforcement.

From equations (3) and (4) we find:

$$f_{b1} = \frac{f_{b2} + \dfrac{\Delta N_x}{A_b}}{\sin\alpha\,(\cot\alpha + \cot\theta)} \tag{5}$$

It will be seen that for common values of α and θ, the denominator in (5) exceeds unity. This means that shear stresses must be transmitted between the reinforcement and concrete over the distance x (i.e. $\Delta N_x > 0$), in order to have $f_{b1} = f_{b2}$. However, as f_{b2} cannot exceed the yield stress f_{yb}, it will be seen that yielding cannot be directly expected to occur in the bent-up bars.

In cases in which there is no significant transverse reinforcement in the form of stirrups over the distance x, only small shear stresses can be transmitted between the concrete and the longitudinal reinforcement here, i.e. $\Delta N_x \cong 0$. Thus, the maximum value of f_{b1} is found to be

$$f_{b1} = \frac{f_{yb}}{\sin\alpha\,(\cot\alpha + \cot\theta)} \tag{6}$$

In the general case, $\alpha = 45°$, $\theta = 45°$, we get

$$f_{b1} = \frac{f_{yb}}{\sqrt{2}} \tag{7}$$

which in turn means that the contribution of the bent-up bars to the shear strength calculated by the traditional method - given by equations (1) and (2) - is overestimated by about 41%.

For uniformly distributed bars we find from (2) and (6) that the contribution of the bent-up bars to the shear strength is

$$V_b = A_b f_{yb} \frac{z}{s} = r_b f_{yb} \sin\alpha \, b_w z \qquad (8)$$

from which it will be seen that the contribution of the bent-up bars is independent of the angle θ of the diagonal crack and the angle α of the bent-up bars.

It will be seen from equation (6) that when $\theta > 45°$, yielding in the bent-up bars can be achieved, e.g. when $\alpha = 45°$ and $\theta = 67.5°$, $f_{b1} = f_{yb}$. However, the number of bent-up bars across the diagonal plan decreases with increasing values of θ, so that, as mentioned above, the total contribution to the shear strength becomes independent of both α and θ.

The above considerations apply near the point of bending-up. However, the value of f_{b1} obtained from (6) is also the maximum stress for which we must calculate in cases where a diagonal crack crosses the bar after the bending-up point. If the stress in the bar exceeded this value higher up in the web, then shear stresses would have to be transmitted between the reinforcement and the concrete over the intervening distance. However, such shear stresses would produce bending in the concrete web here, so these shear stresses would in this case be limited by the strength of the concrete lamella.

3. *TESTS*

In the formulation of the above theory - equations (6) to (8) - it is assumed that only small shear stresses are transmitted between the concrete and the straight part of the longitudinal reinforcement over the distance x (fig. 2), i.e. $\Delta N_x \cong 0$. The validity of this assumption is investigated in the following on the basis of tests on beams in which the shear reinforcement consists exclusively of bent-up bars or of a combination of bent-up bars and vertical stirrups.

On the other hand, tests in which the shear reinforcement consists only of e.g. inclined stirrups cannot be used to check the theory because in these cases, the longitudinal reinforcement is continuous, whereby different equilibrium equations apply.

3.1 Özden's Tests

In tests carried out by Özden [11] at the Structural Research Laboratory, Technical University of Denmark, the development of strain in the bent-up bars was measured by means of strain gauges. Fig. 3 shows the results of these measurements on three beams with different shear reinforcement. Beam T9, which was reinforced solely with stirrups, is included for the purposes of comparison. Fig. 4 shows the crack pattern in the three beams just before failure, and table 1 indicates the degree of the shear reinforcement and the angles of the diagonal cracks θ_{test} measured in the tests. The table also gives the value of the ratio f_{b1}/f_{b2} calculated on the basis of the present theory (α and θ_{test} inserted in equation (6)).

Beam No.	Type of★ web reinforcement	α	f_{yb} f_{ys} MN/m²	$\Sigma K r_s f_{ys}$★★ or $\Sigma r_b f_{yb} \sin \alpha$ MN/m²	θ_{test}	$\frac{f_{b1}}{f_{b2}}$	
						theory	test
T9	6R8	90°	278	14.5	~ 45°	-	-
T12	2K14 1K16	45° 45°	439 443	19.4	40-50°	0.6-0.8	0.4-0.7
T13	2K14 1K16 4R5	45° 45° 90°	439 449 300	19.5 4.1	50-60°	0.8-0.9	0.9-1.0

★ R8: stirrups, plain, round, diam. = 8 mm.
K14: bent-up bars, deformed (»Welbond»), diam. = 14 mm.

★★ Summation across total shear span a, $r_s = \Sigma A_s/ab_w \sin \alpha$

Table 1

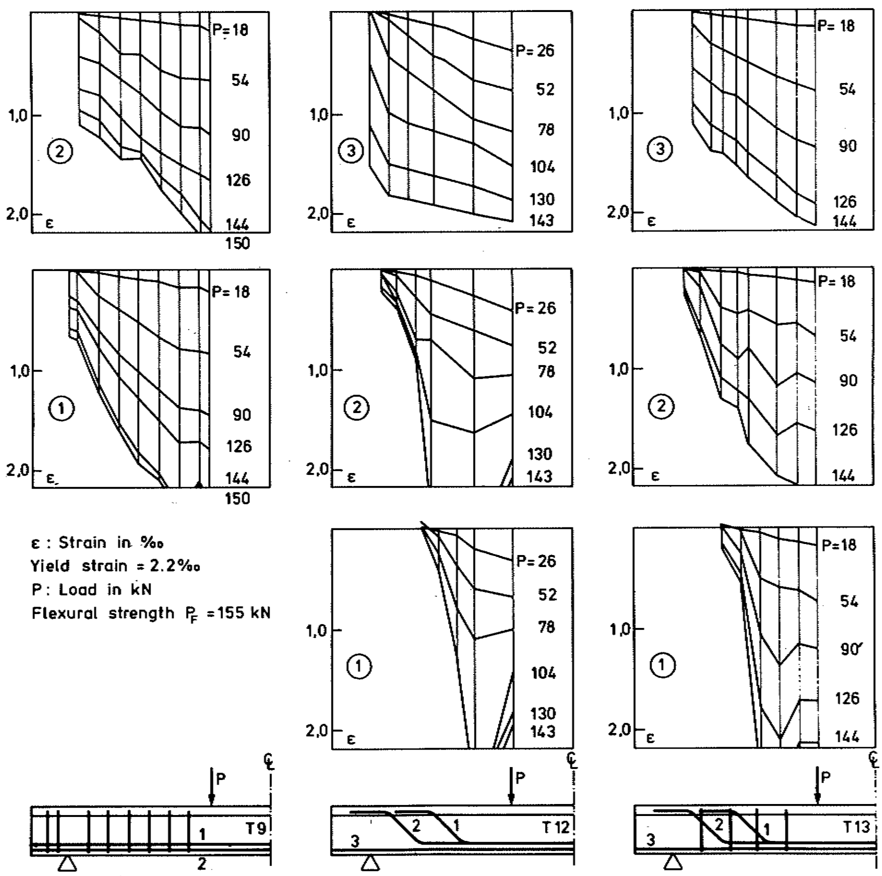

Fig. 3. Strain measurement on the longitudinal reinforcement and the bent-up bars. Tests by Özden [11].

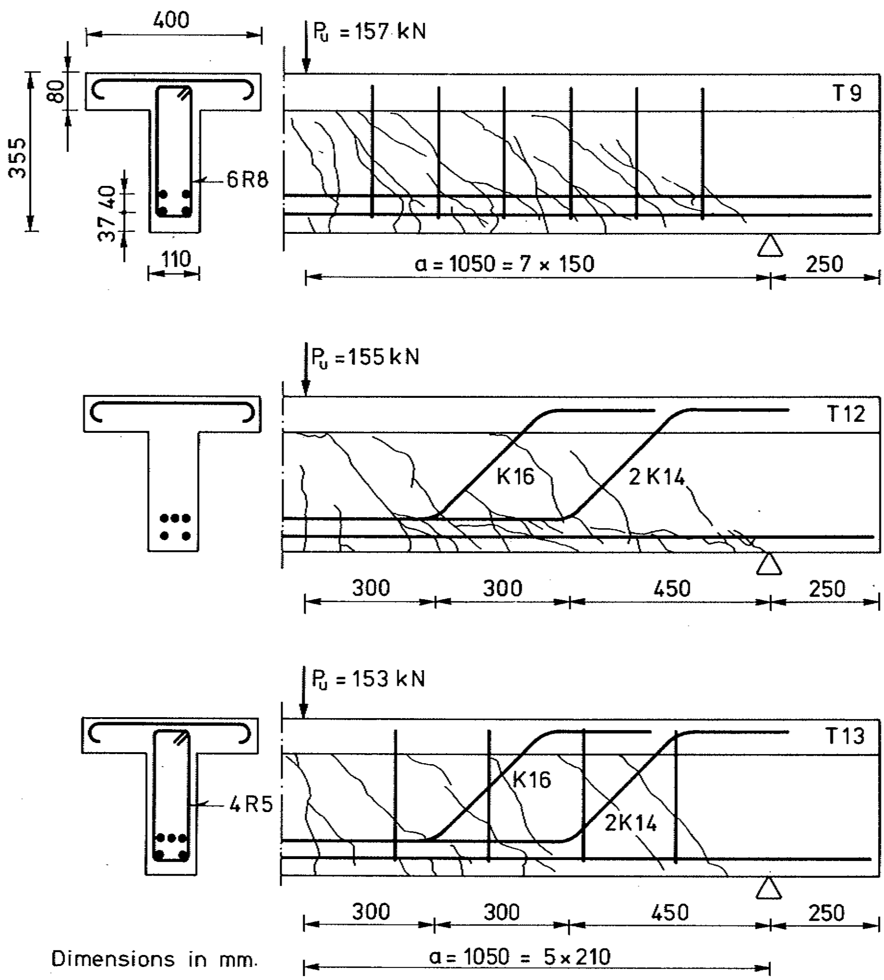

Fig. 4. Crack pattern just before failure. Tests by Özden [11].

As will be seen from fig. 3, in beam T12, yielding occurred in both bent-up bars immediately before the bending-up point, while the stress immediately after this was considerably reduced. The crack angles observed were $\theta_{test} = 40° - 50°$, cfr. fig. 4 and table 1.

In beam T13 the stress immediately before and after the bending-up point was approximately equal to the yield stress. The crack angles observed were considerably larger than in the case of beam T12, viz. $\theta_{test} = 50° - 60°$.

It will thus be seen from the last column in table 1 that the results of the tests are in excellent accordance with the theory.

3.2 Richart's Tests

In an investigation covering a large number of tests, Laupa, Siess, and Newmark [12] have found that the shear strength V_u for beams with shear reinforcement in the form of stirrups ($\alpha = 45°$ and $90°$) can be expressed by

$$V_u = V_0 \left(1 + k \frac{A_s f_{ys}}{s\, b_w \sin \alpha}\right) = V_0 \left(1 + k\, r_s f_{ys}\right) \qquad (9)$$

where

A_s: cross-sectional area of the stirrups,

f_{ys}: yield stress in the stirrups,

k: a constant,

V_0: a function of the compressive strength of the concrete, the degree of longitudinal reinforcement and the beam cross-section.

The regression line corresponding to equation (9) was centrally located in the test material investigated.

Following this, these authors investigated the shear strength of beams with bent-up bars (tests by Richart) and depicted V_u/V_0 as a function of $r_b f_{yb}$, as shown in fig. 5a. Their conclusion is that the regression line (9) can also be used in this case, even though it is not located centrally in the test material.

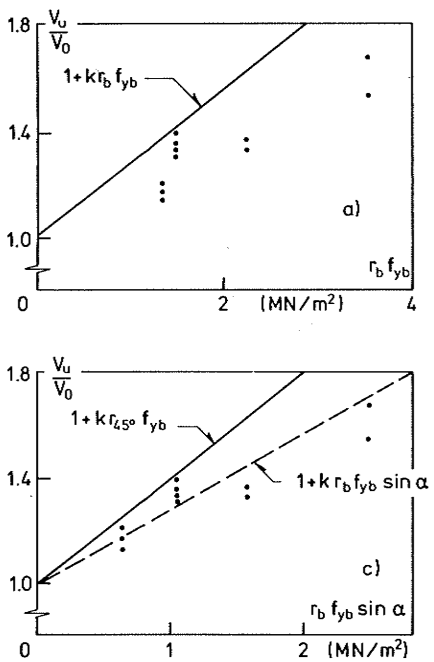

Fig. 5. Relationship between shear strength and degree of bent-up bars. Richart's tests.
a) according to [12],
b) according to [12] with correction for K,
c) according to [12] with correction necessitated by theory under consideration (equation (11)).

$$v_u = \frac{V_u}{b_w z} \qquad (12)$$

where the lever arm z is calculated corresponding to the ultimate state analysis of the cross-section with maximum bending moment (for the present case z = 0.90 h).

Fig. 6b depicts v_u as a function of the contribution of the shear reinforcement to the shear strength, calculated in the traditional manner and taking the K-factor into account. A comparison with fig. 6a, in which the K-factor is neglected, shows that, as mentioned earlier, the K-factor must be taken into account in order to describe the degree of shear reinforcement when α is not 45° and 90°. It will further be seen from fig. 6b that v_u increases more heavily than found by Mörsch in tests on beams with stirrup reinforcement - as has also been observed in a number of other tests. On the other hand, it will be seen that for beams reinforced with stirrups combined with bent-up bars, there is a smaller increase in v_u than obtained by the traditional Truss Analogy method of calculation. The slope of the curve through the test results is approximately $1/\sqrt{2}$ that of the slope given by Mörsch (equation (2a)).

In fig. 6c, v_u is depicted as a function of the degree of shear reinforcement calculated in accordance with the theory under consideration (equation (8)). It will be seen that the results of the tests with a combination of bent-up bars and stirrups are in excellent accordance with the theory, the curve through the test results for beams with bent-up bars running parallel to that resulting from the theory.

4. DISCUSSION

In the above it has been shown that the contribution of bent-up bars to the shear strength is overestimated, when the traditional method of calculation (Mörsch) is used. Although this method has been used for more than 60 years, the author knows of no reports of accidents resulting from this. However, this is presumably due to the fact that the methods of calculation for shear specified in Codes of Practice result in bearing capacities that considerably exceed the actual ultimate shear force to which the beams are subjected. Nevertheless, as this reserve capacity is gradually being reduced by the formulation of more correct calculations, mainly based on tests on beams with stirrup reinforcement there is an increasing risk of accidents as a result of this incorrect evaluation. In a number of countries, this risk is being gradually reduced, because the use of bent-up bars as shear reinforcement is decreasing, partly because this type of reinforcement entails the risk of undesirable cracking at the point of bending-up and partly because it is considerably more laborious than shear reinforcement in the form of stirrups.

It may seem surprising that no one except K. W. Johansen has earlier drawn attention to the fact that the usual method of calculating bent-up bars is erroneous. The reason for this is probably partly that most laboratory tests on this type of reinforcement have been aimed at investigating whether calculations in accordance with the various theories result in a sufficient shear strength for beams, cfr. e.g. Leksukhum and Smith [16] for the latest tests of this type, and partly that astonishingly few of the more than a thousand shear tests carried out over the years have been directly aimed at studying only the effect of the degree and type of shear reinforcement on the shear strength.

5. CONCLUSIONS

Theoretical considerations based on the diagonally cracked beam have shown that the contribution of the bent-up bars to the shear strength should be calculated as

$$V_b = r_b f_{yb} \sin\alpha \, b_w z = A_b f_{yb} \frac{z}{s} \qquad (8)$$

from which it will be seen that the contribution is independent of the angle θ of the diagonal crack and the angle α of the bent-up bars with the axis of the beam.

This method of calculation is now specified in the Code of Practice for the new Danish Concrete Standards.

The validity of the theory is confirmed by existing tests, partly by means of strain measurements on the reinforcement and partly by means of investigations of the dependence of the ultimate shear strength on the degree of web reinforcement. For the most common cases in practice, this means that the use of the traditional method of calculation (Mörsch) results in an approximately 41% overestimation of the contribution of the bent-up bars to the shear strength.

In the case of the latter type of reinforcement K. W. Johansen [7] came to the same conclusion in 1928, although by a different method. However, this view has won very little recognition outside Denmark.

6. ACKNOWLEDGMENT

I am indebted to Dr. P. E. Regan, Imperial College, University of London, for his kind assistance in sending the original reports on the tests performed by Khan and Placas.

7. REFERENCES

[1] ACI-ASCE Committee 326. *Shear and Diagonal Tension - Part 2, Beams and Frames.* ACI-Journal, Proceedings V. 59. No. 2, Feb. 1962, pp 277-334.

[2] LEONHARDT, F., WALTHER, R. *Schubversuche an Plattenbalken mit unterschiedlicher Schubbewehrung. (Shear Tests on T-beams with Varied Type of Web Reinforcement).* Deutscher Ausschuss für Stahlbeton, (Berlin), H. 156, 1963, 84 pp.

[3] LEONHARDT, F. *Reducing the Shear Reinforcement in Reinforced Concrete Beams and Slabs.* Magazine of Concrete Research (London), V. 17, No. 53, 1965, pp. 187-198.

[4] FERGUSON, P. M., MATLOOB, F. N. *Effect of Bar Cutoff on Bond and Shear Strength of Reinforced Concrete Beams.* ACI-Journal, Proceedings V. 56. No. 1, Sep. 1959, pp. 5-24.

[5] LEONHARDT, F., WALTHER, R. *Versuche an einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung, (Tests with Reinforced Concrete Beams with and without Web Reinforcement).* Deutscher Ausschuss für Stahlbeton (Berlin), H. 151, 1962, 83 pp.

[6] RICHART, F. E. *An Investigation of Web Stresses in Reinforced Concrete Beams.* University of Illinois, Engineering Experiment Station. Bulletin No. 166, 1927, 106 pp.

[7] JOHANSEN, K. W. *Om Virkningen af Bøjler og Skraajærn i Jærnbetonbjælker. (The Effect of Stirrups and Bent-up Bars in Reinforced Concrete Beams).* Danmarks Naturvidenskabelige Samfund (Copenhagen). A. No. 17, 1928, 50 pp.

[8] JOHANSEN, K. W. *Beregning af Jernbetonbjælker. (Calculation of Reinforced Concrete Beams).* Bygningsstatiske Meddelelser (Copenhagen), V. 16, No. 2, 1945, pp. 33-63.

[9] JOHANSEN, K. W. *Critical Remarks on the Effect of Bent-up Bars and Stirrups in Reinforced Concrete Beams.* 5th IABSE Congress Lisbon 1956, Final Report (1957) pp. 507-512.

[10] SØRENSEN, H. C. *Forskydningsarmering i jernbetonbjælker. (Web Reinforcement in Reinforced Concrete Beams).* Structural Research Laboratory, Technical University of Denmark, Report No. 19, 1970, 122 pp.

[11] ÖZDEN, K. *An Experimental Investigation on the Shear Strength of Reinforced Concrete Beams.* Technical University of Istanbul, 1967, 249 pp.

[12] LAUPA, A., SIESS, C. P., NEWMARK, N. M. *Strength in Shear of Reinforced Concrete Beams.* University of Illinois, Engineering Experiment Station, Bulletin No. 428, 1955, 73 pp.

[13] KHAN, M. H. *Shear Strength of Reinforced Concrete Beams Reinforced with Bent-up Bars.* M.Phil. Thesis, Imperial College, University of London, 1969, 209 pp.

[14] PLACAS, A. *Shear Failure of Reinforced Concrete Beams.* Ph.D. Thesis, Imperial College, University of London, 1969.

[15] REGAN, P. *Shear in Reinforced Concrete - an Experimental Study.* Imperial College, University of London, 1971, 203 pp.

[16] LEKSUKHUM, K., SMITH, R. B. L. *Comparative Study of Bent-up Bars with other Forms of Secondary Reinforcement in Beams.* ACI-Journal, Proceedings V. 68, No. 1, Jan. 1971, pp. 32-35.

8. SUMMARY

This article describes a theoretical investigation of the contribution of bent-up bars to the shear strength of reinforced concrete beams, which shows that the contribution is independent of the inclination of the bent-up bars and the angle of the diagonal crack. This is at

variance with the traditional Truss Analogy and means that the contribution calculated according to this theory is overestimated by about 41%. The present theory has been compared with tests and is in exact agreement with these.

The theory is an extension of a theory presented by K. W. Johansen as early as 1928. This theory seems not to be accepted outside Denmark.

9. RESUMÉ

I denne artikel fremlægges en teoretisk undersøgelse af opbøjet længdearmerings (»skråjerns») bidrag til armerede betonbjælkers forskydningsbæreevne. Teorien viser, at dette bidrag er uafhængig af den opbøjede længdearmerings hældning og hældningen af det diagonalrevnede snit. Dette er i modstrid med den traditionelle gitteranalogi og medfører, at den opbøjede armerings bidrag til forskydningsbæreevnen ved anvendelse af gitteranalogien overvurderes med ca. 41%.

Teorien er sammenlignet med en række forsøg, og den er fundet i god overensstemmelse hermed.

Den fremlagte teori er en udvidelse af en teori fremlagt af K. W. Johansen allerede i 1928. K. W. Johansens teori synes imidlertid ikke at have vundet megen anerkendelse uden for Danmarks grænser.

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