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DYNAMIC PROPERTIES OF ANTI-VIBRATION MOUNTINGS

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Dynamic Properties of Anti-Vibration Mountings

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ABSTRACT

A forced vibration non-resonance technique is described for the measurement of elastic modulus and damping factor of rubber-mountings.

Some representative results are obtained in the frequency range up to 1 kHz, and further the effects of various parameters as temperature, mean force level and force amplitude are considered.

It is also shown how the values of the elastic modulus and damping factor can be used to predict the transmissibility of the mountings.

INTRODUCTION

In factories, where the requirements for comfort for the workers in the last years have become greater, it has been common to insulate heavy machinery from the buildings, in order to prevent transmission of vibrations.

For this purpose many types of resilient mountings have been developed. For most of the products, the information given by the manufacturers is sparse and so it is difficult for a consulting engineer to make a proper choice of bearings suitable for a certain task.

It is shown how information about the dynamic quantities could be presented in order to facilitate the design of resilient bearings.

1. THEORY

Fig. 1 shows the mounting placed in the testing position and acted upon by the harmonic force $\tilde{P} = P \sin \omega t$. The force in the mounting is denoted $\tilde{K}$, and the displacement of the mass m is denoted $\tilde{u}$.

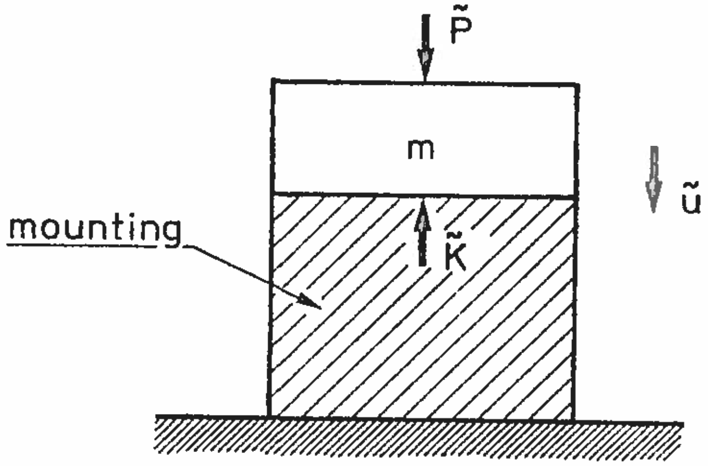

Fig. 1. Mounting on rigid foundation.

The following complex representation is used (see for instance Snowdon [1]):

$$\tilde{P} = \mathrm{Re}\left[P \; e^{i\omega t} \right]$$

$$\tilde{K} = \mathrm{Re}\left[K^{*} \; e^{i\omega t} \right] \qquad (1)$$

$$\tilde{u} = \mathrm{Re}\left[u^{*} \; e^{i\omega t} \right]$$

where *denotes a complex quantity, and $i = \sqrt{-1}$.

The complex modulus C^* for the mounting is defined by:

$$K^* = C^* u^* \qquad (2)$$

and C*may be written:

$$C^* = C(1 + i\delta) \qquad (3)$$

where

C = Elastic Modulus.

δ = Damping factor.

The equation of motion for the system of fig. 1 is now:

$$m \frac{d^2\tilde{u}}{dt^2} = \tilde{P} - \tilde{K} \qquad (4)$$

From (4) and (1) follows:

$$-m\omega^2 u^* = P - K^* \qquad (5)$$

On fig. 2, eq. (5) is visualized in the Complex Plane, and the figure should also be used to show the definition of phase angles Θ and Θ'.

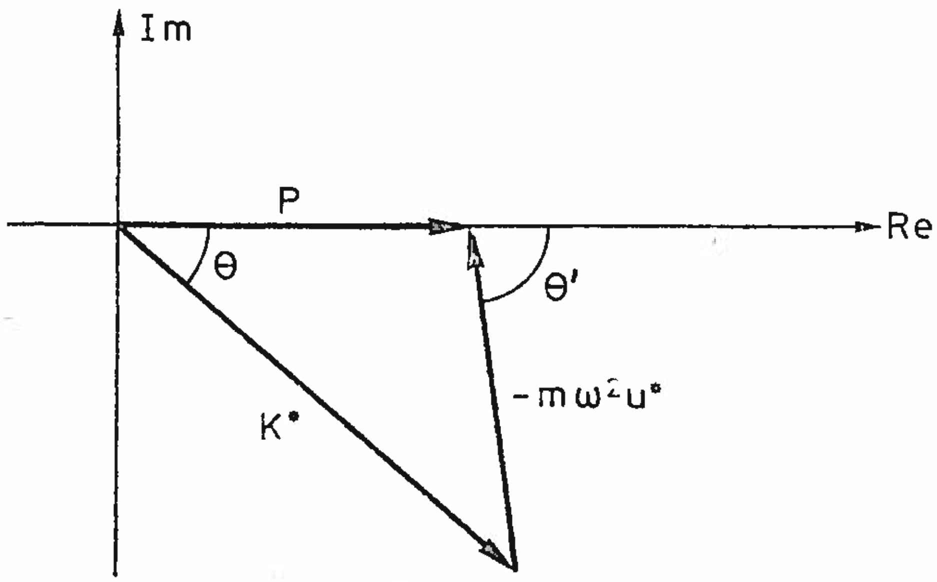

Fig. 2. Vector-representation of dynamic eqation.

Expressions for the modulus and damping are now derived from eq. (2), (3),(5) and fig. 2:

$$C^* = \frac{K^*}{u^*}$$

$$C + iC\delta = \frac{P + m\omega^2 u^*}{u^*} = \frac{P}{u^*} + m\omega^2 \qquad (6)$$

As

$$u^* = |u^*| \ (\cos \Theta' - i \sin \Theta'),$$

and

$$|u^*| = \frac{|a^*|}{\omega^2}$$

where

$$|a^*| = \text{Acceleration amplitude},$$

the result for the modulus is:

$$\underline{C = \omega^2 \ (\frac{P \cos \Theta'}{|a^*|} + m)} \qquad (7)$$

From equation (2) follows

$$K^* \ e^{i\Theta} = C^* u^* \ e^{i\Theta}$$

Fig. 2 shows that

$$K^* \ e^{i\Theta} = |K^*|$$

and

$$u^* \ e^{i\Theta} = |u^*| (\cos(\Theta' - \Theta) - i \sin (\Theta' - \Theta))$$

These expressions show that

$$(1 + i\delta)(\cos(\Theta' - \Theta) - i \sin(\Theta' - \Theta))$$

must be real. Hence

$$\underline{\delta = \tan(\Theta' - \Theta)} \qquad (8)$$

As the last quantity, the transmissibility T of the mounting is defined according to ref. [1] :

$$T = \frac{|K^*|}{P} \qquad (9)$$

and by using eq (2) and (5) the result is:

$$T = \frac{\sqrt{1 + \delta^2}}{\sqrt{(1-\frac{m\omega^2}{C})^2 + \delta^2}} \qquad (1o)$$

Thus the transmissibility can be predicted if both the elastic modulus and the damping factor of the mounting are known.

On the other hand, the C- and δ functions expressed by (7) and (8) can be determined by "measuring" the triangel of fig. 2 for various values of ω.

2. TEST SET-UP AND MEASURING SYSTEM.

Fig 3 shows the measuring set-up for determining the triangel of fig. 2.

A forced vibration non-resonance technique was used, and the mounting was placed with one end against a rigid support and the other end attached to a vibrator (se fig. 3).

By adjusting the support the mean force level could be varied and by the vibrator the force amplitude was controled. By measuring the peak values P and $|K^*|$ of the forces on both sides of the mounting, the peak value $|a^*|$ of the accelera-tion of the vibrating mass and the phase angles Θ and Θ' be-tween these quantities, the modulus C and the damping factor δ could be determined by the equations in section 1.

Most of the apparatus used is manufactured by Brüel and Kjær, but also a Lock-in Amplifier from Princeton Applied Re-search and an oscilloscope from Tectronix were used.

The Vibration Exciter acts upon the mounting through three force-transducers and the signals from transducers and accelerometer are sent through preamplifiers to the "vi-bration-meters". As fig. 3 shows, the forces are read on the Exciter control and the acceleration on the voltmeter. The phase angles between the quantities are measured on the Lock-in Amplifier.

The reason for including the oscilloscope in the system is twofold. The first one is to have a visual control on the phase angles and the magnitudes of the signals. The other one is to utilize the facility of the type of preamplifier used for measuring the force in the mounting. This pre-amplifier, type "2628", is able to measure a very low-frequen-cy signal i.e. a static signal. This means that by this fa-cility and the DC-input on the oscilloscope, the static for-ce in the mounting can be measured.

The Exciter is controled by the Exciter Control via the Power Amplifier in a servo-loop, so that the forceamplitude can be given a constant value. For a test like the present where the natural-frequency is passed during the test, the exciting force cannot be kept constant, while it would damage the mounting . So it was chosen to keep the force amplitude

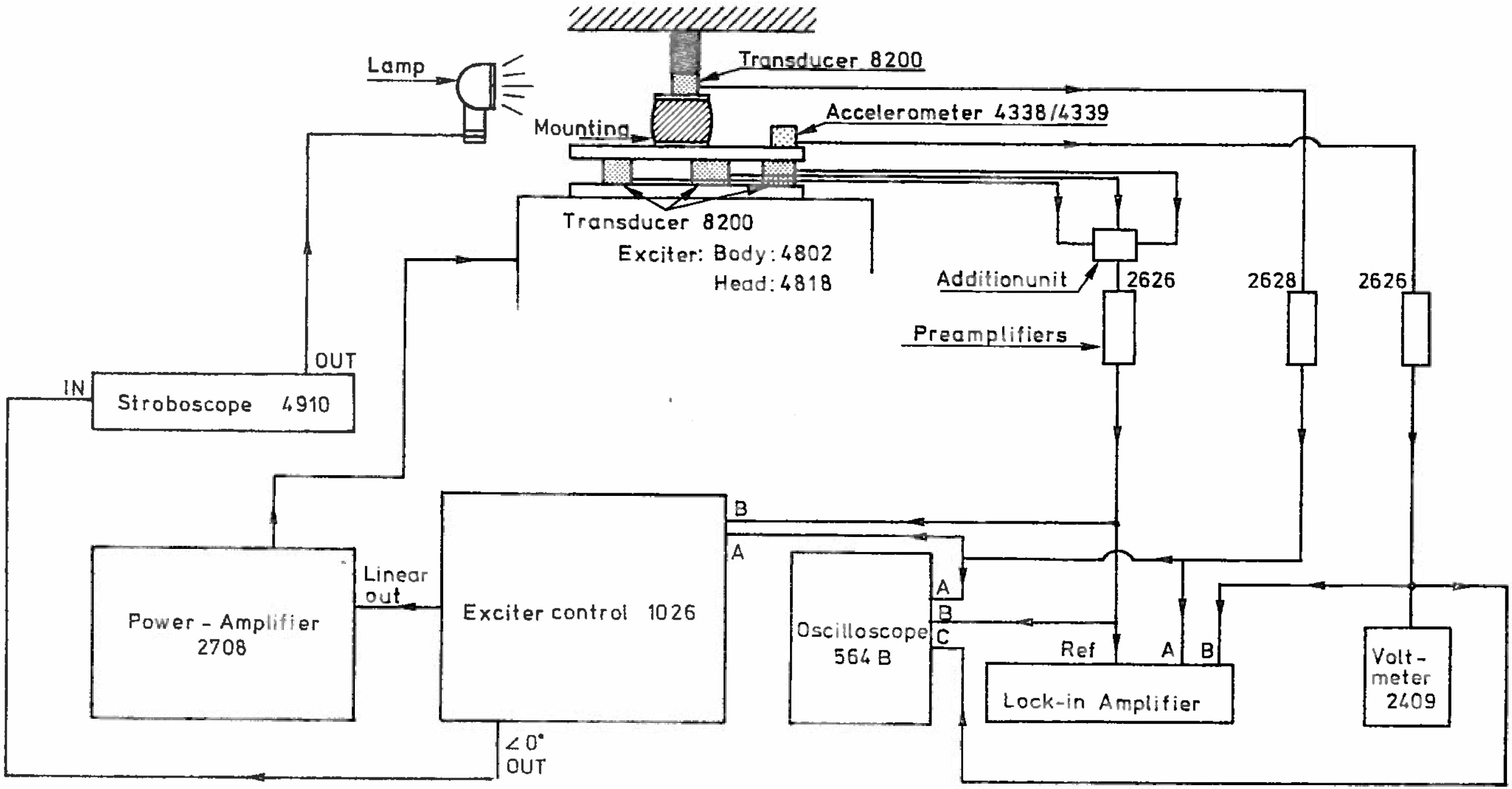

Fig. 3. Measuring Instrumentation Set-up.

$|K^*|$ in the mounting constant for frequencies below the resonance, and to keep the amplitude P of the exiting force constant above resonance.

The Stroboscope is not a part of the measuring system, but has been used to get an impression of the movement of the mounting.

It was not possible to vary the temperature during the tests, so the measurements were taken at temperatures from $24^{o}C$ to $27^{o}C$. However the tests indicated that the temperature had only a slight influence on the measured dynamic quantities in the ranges of temperature and frequency here concerned.

3. RESULTS

Two types of mountings have been tested, namely one type where the rubber is used in compression, and an other where the rubber is used in shear, and both types have only one rubber-body. The types tested are distributed by IKAS-ISOLERING (see ref. [4]), and the following tabel gives some of the information found in the sales catalogue:

Type	No.	Name	Rubber	Load	Max Load
				N	N
	2 3 4	AD 3o25 W AD 4o3o W AD 3o25 R	White White Red	35o 65o 45o	5oo 1ooo 65o
	5 6	MDF 8o4oW MDF 8o4o R	White Red	1oo 15o	15o 25o

Table 1. Mounting data.

According to the catalogue "WHITE" means a soft type and "RED" means a medium type natural rubber.

A. Results for compression-mountings:

Fig. 4 shows the variation of the elastic modulus C with frequency for mountings 2,3 and 4.

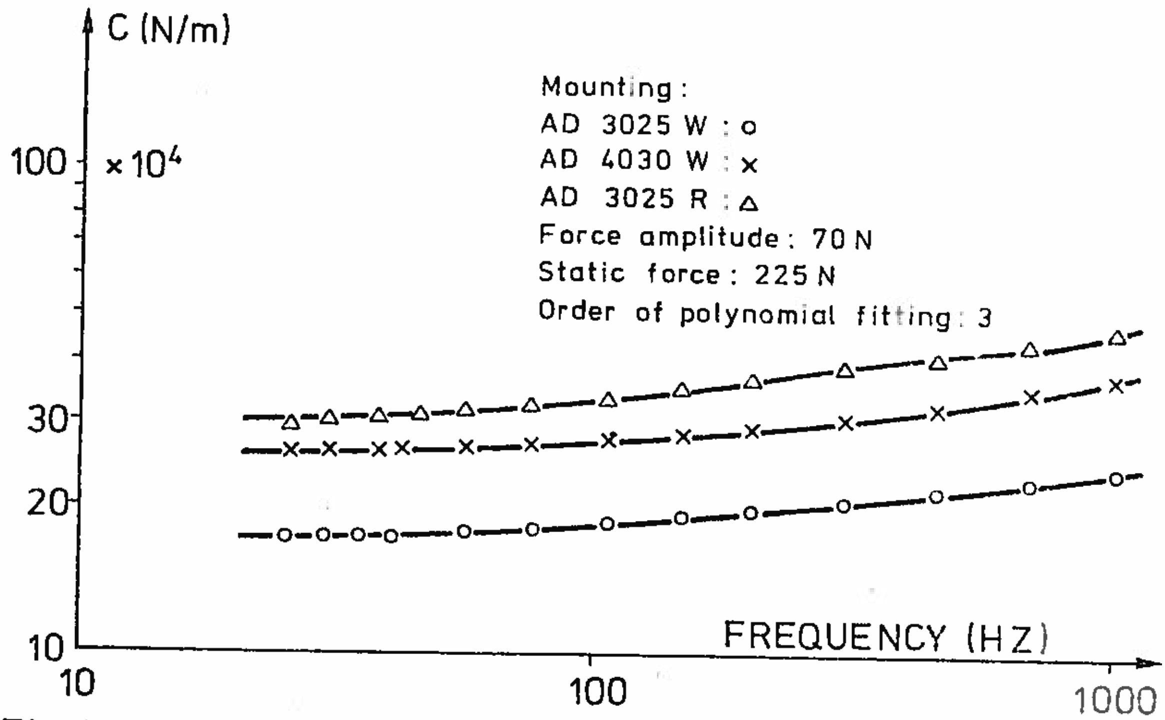

Fig. 4. Elastic Modulus for mounting no. 2,3 and 4.

Fig. 5 shows how the damping factor δ varies with frequency for the same mountings.

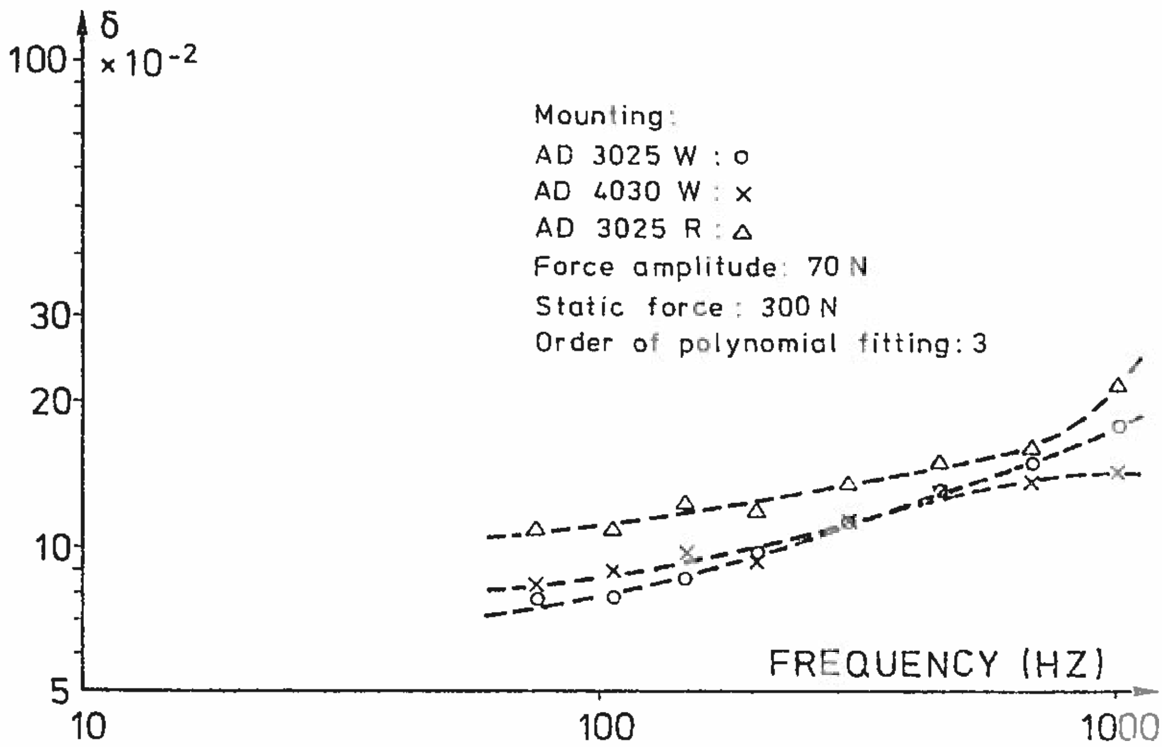

Fig. 5. Damping factor for mounting no. 2,3 and 4.

To illustrate the influence of the previous mentioned parameters: static force and force amplitude, the results from tests with mounting no. 3 are presented on figs. 6, 7, 8 and 9.

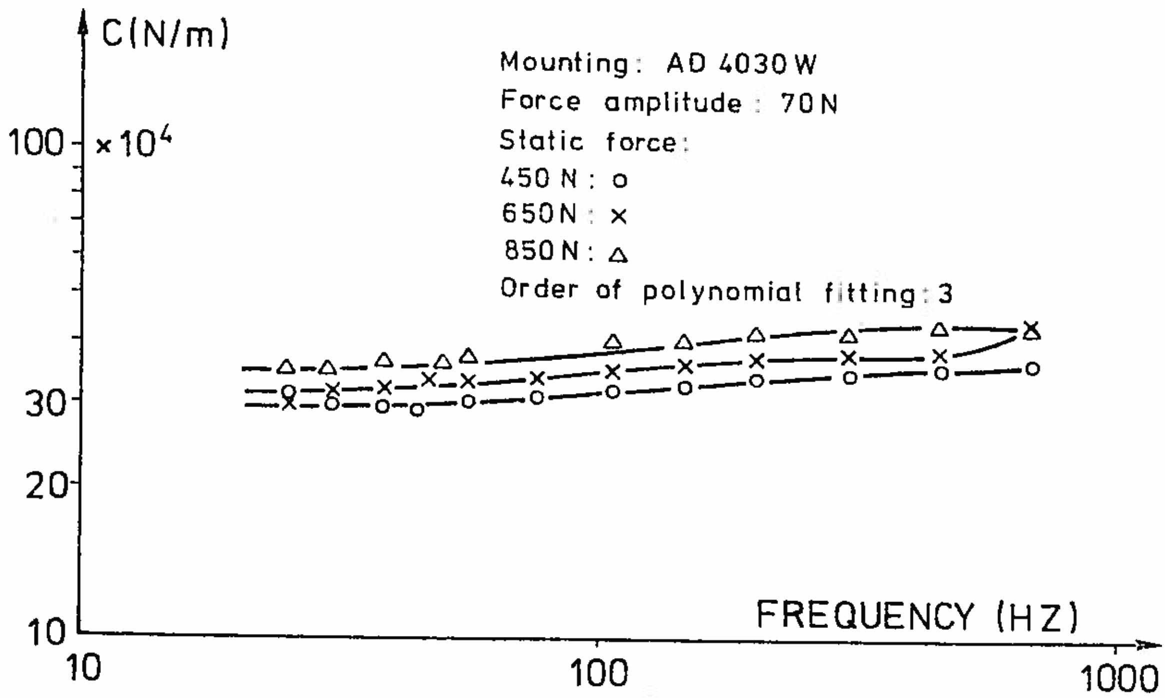

Fig. 6. Elastic Modulus. Constant amplitude. Varying static force.

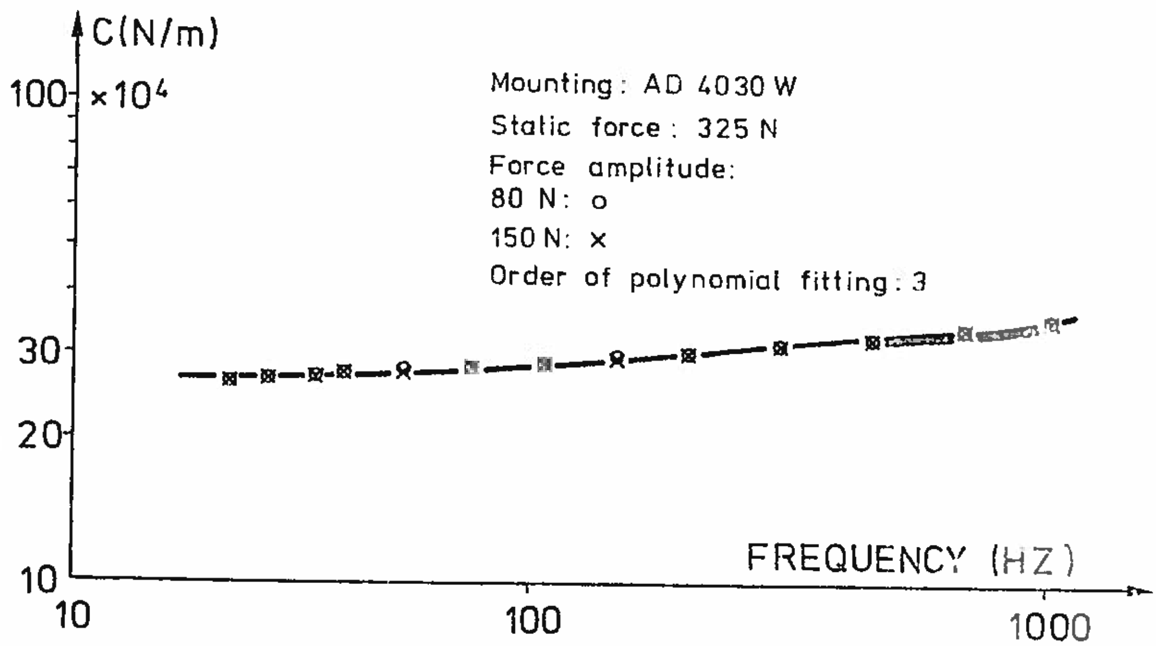

Fig. 7. Elastic Modulus. Constant static force. Varying amplitude.

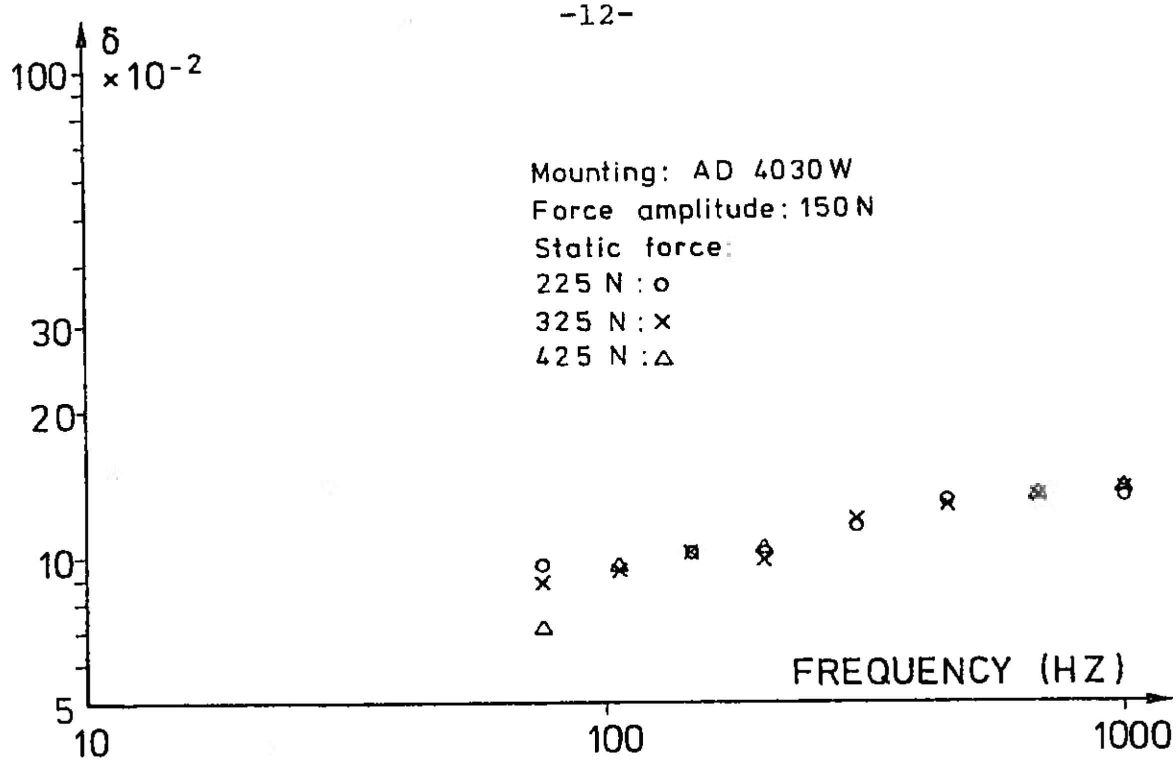

Fig. 8. Damping factor. Constant amplitude. Varying static force.

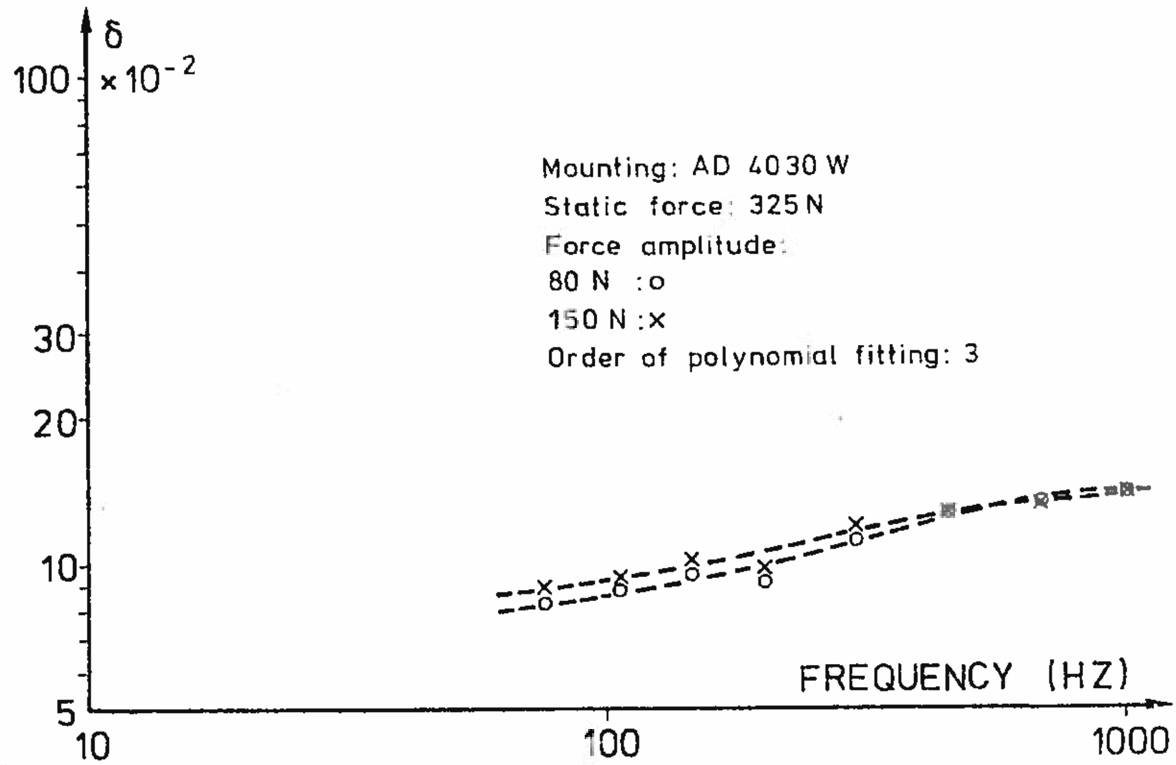

Fig. 9. Damping factor. Constant static force. Varying amplitude.

Fig. 4 shows that the elastic modulus of a mounting depends both upon geometrical shape and rubber-type.

Fig. 5 shows that the damping-functions for the two mountings with the soft rubber are nearly the same, and therefore it seams as if the damping is rather independent of geometrical shape. The damping for the medium rubber-type seams to be higher than for the soft type.

Fig. 6 shows that the modulus is increasing when the static force increases. This is due to the increasing deformation of the mounting. (see fig. 1o).

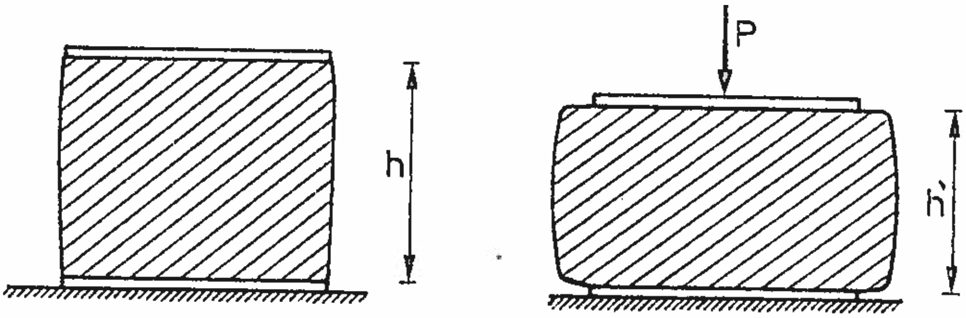

Fig. 10. Deformation of compressionmounting.

Fig. 7 shows that C is independent of the force amplitude.

Fig. 8 shows that the damping factor δ is rather independent of the static force level and from fig. 9 follows that the damping is only slightly dependent upon the force amplitude, most for lower frequencies.

From this last mentioned dependence follows, that the δ-curve cannot be represented in the whole tested frequency-range, because the amplitude of the exiting force cannot be kept constant in the whole range. As the damping seems to depend upon the force amplitude the change in this at the natural frequency would "disturb" the functions plotted.

As the elastic modulus is independent of amplitude, the same precaution is unnecessary for this quantity.

All the figures show that the C- and δ-functions depend upon the frequency in such a way, that increasing frequency gives an increase in both C and δ.

B. Results for shear-mountings:

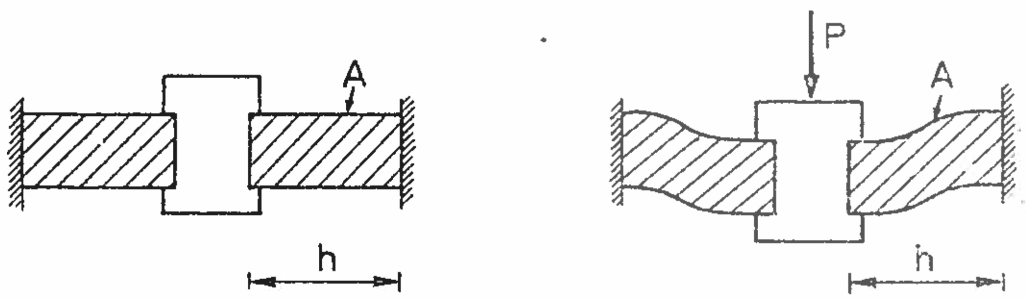

Fig. 11. Deformation of shearmounting.

Fig. 12 shows the elastic modulus for the two shear-mountings: no. 5 and 6.

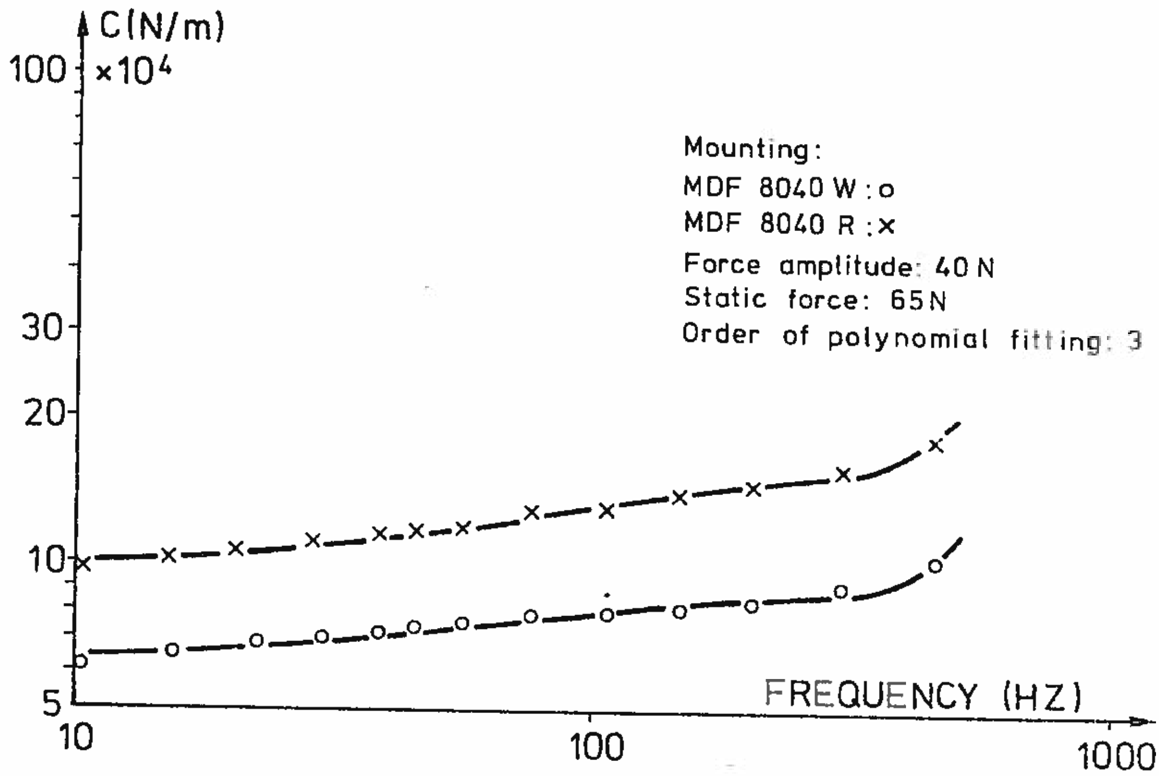

Fig. 12. Elastic Modulus for shear mountings.

If these results are compared with fig. 4, it can be seen, that C for the compression-type with the same rubber-type is about three times C for the shear-type. This is in agreement with the fact, that for rubber:

$E = 3\ G$

where

E = Compression modulus

G = Shear modulus

On fig. 13 the damping factor is plotted, for the same two mountings, and as for the compression-type, the damping for the medium type of rubber seems to be a little higher than for the soft type.

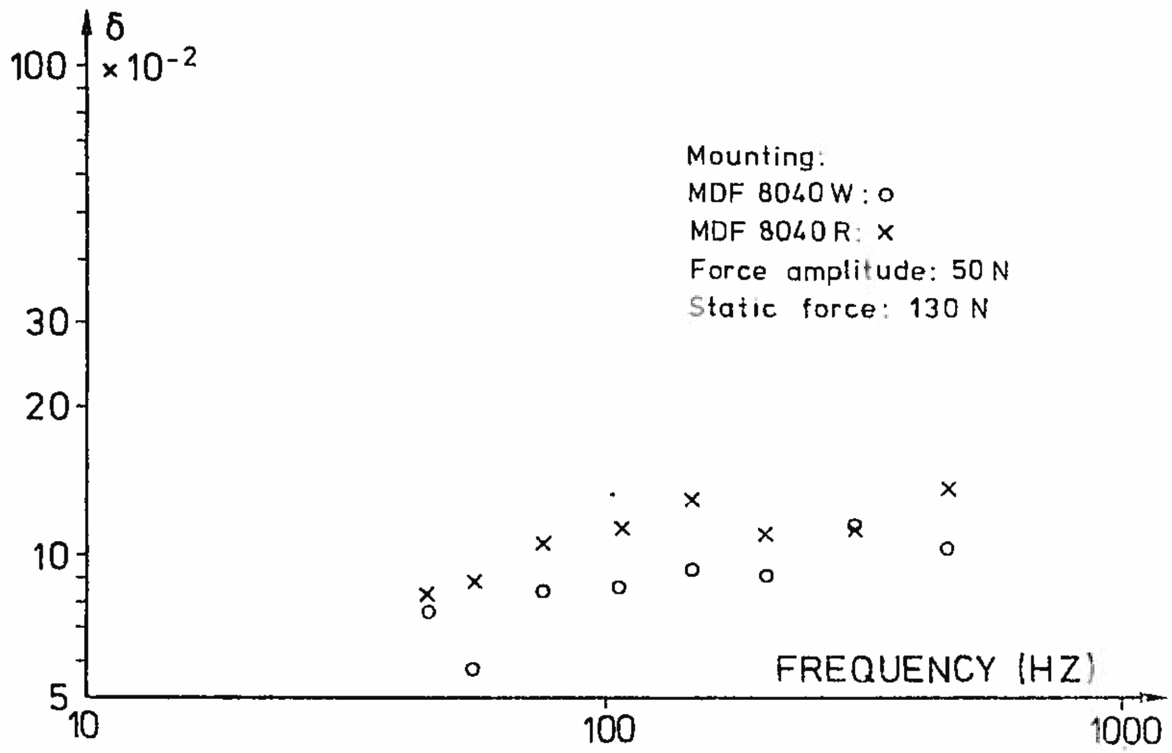

Fig. 13. Damping factor for shear-mountings.

From the figure compared with fig. 5 is also seen, that δ is the same for the compression-type and the shear-type mounting. This indicates that δ depends only upon the rubber-type.

For the shear-type mountings the influence of the parameters: static force and force amplitude have also been examined.

As for the compression-type it was found that the damping δ was independent of static force and only slightly dependent of force amplitude, most for lower frequencies.

Also, as for the compression mountings, it was found that the elastic modulus C was independent of the amplitude.

But in contradiction to the compression-type it was found, that for the shear-type, C was independent of the static force (see fig. 14).

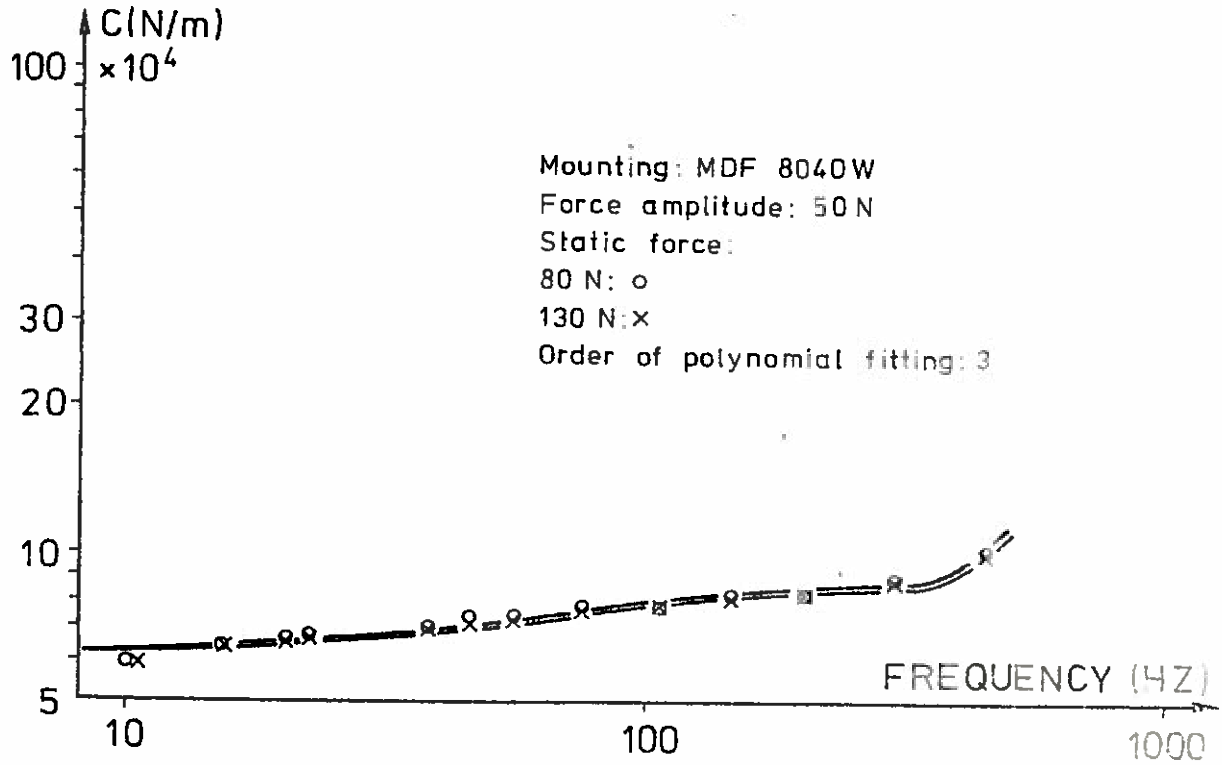

Fig. 14. Elastic Modulus for shear-mounting. Varying static force.

The reason for this seems to be that, as shown in fig. 11, the "shear-area"A and the "height" h are both independent of the static force.

4. CONSLUSION

The measuring system used seems to give good results for the dynamic quantities of the mountings. And as shown in section 1, one is able to predict the transmissibility across a mounting when C and δ are known.

To show this, both the model:

$$T = \frac{\sqrt{1 + \delta^2}}{\sqrt{(1 - \frac{m\omega^2}{C})^2 + \delta^2}}$$

and the measured quantity:

$$T = \frac{|K^*|}{P}$$

are plotted in fig. 15 for a compressionmounting. A good agreement between model and test is found.

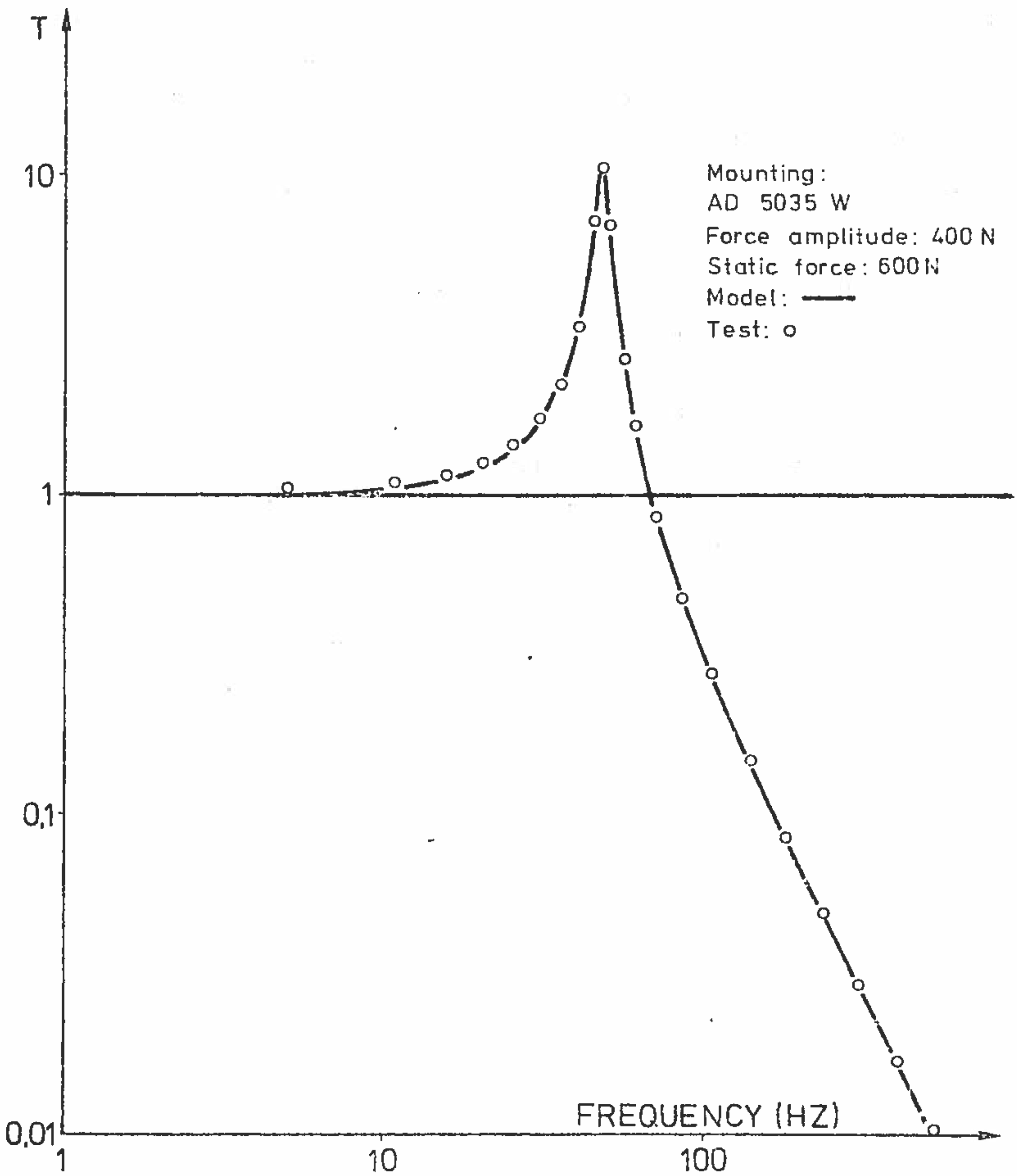

Fig. 15 Transmissibility -curve. Model and test.

However it must be mentioned, that for the tested mountings where the internal damping is small, the phase-meter in use in the measuring set up must be of a very good quality if inaccuracies should not be introduced in determing the damping factor δ, as the phase differences to be measured is of the order of just a few degrees.

The used phase-meter had a standard deviation of about 3^{o}, which was not satisfactory.

A control against gross errors was obtained by using the Least-Square-Method, and this is why five quantitites of the triangel of fig. 2 was measured although it is determined only by three.

The results presented in this paper are just the start of a series of tests with different kinds of resilient mountings,which will be performed in the near future at the Structural Research Laboratory.

NOTATION

The following symbols are used in this paper:

a^* = accelaration amplitude

C^* = complex dynamic modulus

C = dynamic elastic modulus

i = $\sqrt{-1}$

$\tilde{K}$ = harmonic force in mounting

K^* = complex force amplitude

m = vibrating mass

$\tilde{P}$ = harmonic force acting on the mounting

P = force amplitude

t = time

$\tilde{u}$ = harmonic motion of the mass

u^* = complex amplitude of motion

δ = damping factor

ω = frequency

θ, θ' = phase angles

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