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PARTIAL PRESTRESSING

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PARTIAL PRESTRESSING

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CONTENTS

PARTIAL PRESTRESSING

1. SYNOPSIS

Prestressing has a great influence on the behaviour of concrete structures under service loading, but little influence on the ultimate load. Methods of analysis are suggested for partially prestressed structures under service loading, where the method of superposition is not applicable. Partial prestressing with mixed reinforcement offers a number of advantages and is likely to be the most economical solution for medium-span structures.

2. INTRODUCTION

Prestressed concrete techniques, based on the ingenious ideas of E. Freyssinet, have developed rapidly since about 1950. Freyssinet considered prestressed concrete as a new and revolutionary structural material where tensile stresses in the concrete or at least cracks had been prevented. Such fully prestressed concrete was regarded as being basically different from traditional reinforced concrete.

The use of partially prestressed concrete was suggested on various occasions during the 'fifties - in particular by P. W. Abeles - but the Freyssinet group opposed the idea and this view dominated for several years.

During the last decade, however, partially prestressed concrete has received increasing attention.

3. BASIC CONCEPTS

3.1 FULL AND PARTIAL PRESTRESSING

The terms full and partial prestressing refer to the behaviour of the concrete structure under service load. However, different definitions have been suggested, the criterion being the occurrence of one of the following phenomena: cracks; concrete tensile stresses; tensile stresses exceeding a given limit; or closing and reopening of cracks.

In the following, a section will be called fully prestressed if cracks do not develop in the concrete under service loading and will be called partially prestressed if they do.

This definition has the practical advantage that the structural analysis of each of the two types of structures has to be based on individual assumptions, viz. uncracked and cracked cross-section, respectively.

The same definition can refer to a cross-section rather than to the structure as a whole.

If the load exceeds the service load, cracks may develop even in fully prestressed structures. In a partially prestressed section, cracks may develop under service loading. The stress - strain relationship for the concrete is consequently bi-linear and the method of superposition is not applicable in the service load analysis. The sections must be analysed for the relevant combinations of prestressing, service loads, shrinkage, etc.. Methods of tackling this complication are suggested in the following.

3.2 MIXED REINFORCEMENT

Partial prestressing can, in principle, be established by tensioning high-tensile prestressing reinforcement to a moderate stress level, but this solution is not likely to be economical. The most economical way of obtaining a given prestressing force will usually be to provide a sufficient amount of prestressing steel to achieve this force with the prestressing steel fully utilized, i.e. tensioned to its permissible stress.

For partially prestressed structures some additional reinforcement may be required, for instance, with a view to the ultimate limit state of rupture or to limiting crack widths. In many cases this additional reinforcement is most economically provided by means of non-prestressed reinforcement. For this reason partially prestressed structures often have mixed reinforcement, i.e. a combination of prestressed and non-prestressed reinforcement - usually with very different mechanical properties.

3.3 RATIO OF PRESTRESSING STEEL

When a concrete structure with mixed reinforcement is loaded up to its ultimate bending moment, the tensile force N_u in the cracked tensile zone is resisted partly by the prestressing steel (N_{up}) and partly by the non-prestressed steel (N_{us}). The ratio of N_{up} and N_u has been suggested as a measure of what is called the "degree of prestressing". The term is misleading as this quantity is hardly influenced by the stresses introduced by the prestressing.

Here, a slightly modified definition will be used. At the ultimate bending moment, M_u, the lever arms of N_{up} and N_{us}, i.e. their distances from the resultant of the stresses in the compression zone, are z_p and z_s, respectively, and their moments with respect to the point where this resultant intersects the cross-section are:

$$\boxed{M_{up} = z_p N_{up}} \quad \text{and} \quad \boxed{M_{us} = z_s N_{us}}$$

The ratio between the ultimate bending moment resisted by the prestressing steel and the total ultimate bending moment is:

$$\boxed{\vartheta = \frac{M_{up}}{M_u}} \quad \text{where} \quad \boxed{M_u = M_{up} + M_{us}}$$

This quantity varies between zero for reinforced concrete without prestressing steel and unity for concrete reinforced with prestressing steel only.

Like the ratio between N_{up} and N_u, the quantity ϑ does not represent a "degree of prestressing". This is better characterized by the relative prestress χ defined in Section 3.8. However, whereas χ depends on the bending moment under service loading, ϑ does not. This means that ϑ characterizes the structure as such, without reference to the service condition.

The quantity ϑ will be referred to in the following as the ratio of prestressing steel.

According to this definition, the ratio of prestressing steel for a cross-section that must resist both positive and negative bending moments usually depends on the sign of the moment. In case of biaxial bending, the ratio of prestressing steel usually depends upon the inclination of the bending moment vector considered.

3.4 INTERNAL PRESTRESSING FORCES

In general, the forces transferred from the tendons to the concrete create certain internal forces in the structure. These internal forces will be referred to in the following as internal prestressing forces. An internal force is defined here as the resultant of all stresses in a cross-section, including both concrete and steel stresses.

The forces which the tendons transfer to the concrete represent a system of forces in equilibrium.

In a statically determinate beam or frame structure such a system of forces will not create any reactions, and the internal prestressing forces will thus be equal to zero.

3.5 RESIDUAL STRESSES

Any stresses existing in a cross-section for which the internal force is equal to zero are referred to as residual stresses.

The prestressing results in imposed deformation of steel and concrete and consequently in residual stresses.

For statically determinate beam and frame structures the stresses due to prestressing consist only of these residual stresses.

For statically indeterminate structures the effect of prestressing comprises two contributions, viz. the residual stresses and the stresses corresponding to the internal prestressing forces.

If the internal force in a cross-section is equal to zero, the resultant of the residual stresses will be equal to zero. Consequently the resultant of the stresses in the prestressing steel and the resultant of the stresses in the concrete and in the non-prestressed reinforcement must be of the same magnitude, must pass through the same point of the cross-section, must fall on the same line in space, and must be of opposite signs or directions. This is the case with the residual stresses caused by prestressing in statically determinate beams or frame structures, but generally not in statically indeterminate structures due to the internal prestressing forces.

For statically determinate beams and frame structures, the stresses in the concrete and in the non-prestressed steel due to prestressing can thus be calculated by considering the transformed cross-section of the concrete and non-prestressed steel as loaded by a compressive force (Fig. 1 b) of the same magnitude as the prestressing force P (Fig. 1 a) and with the same location and inclination. In general, this is not so in the case of statically indeterminate structures. Nor does the rule apply to slabs and shells because the distribution of bending moments in such structures has a statically indeterminate character.

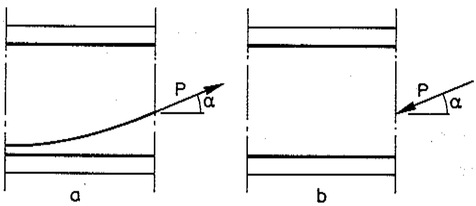


Fig. 1

3.6 ASSUMPTIONS FOR SERVICE LOAD STRESS ANALYSIS

For structures with bonded reinforcement changes in strain are assumed to be proportional to the distance from the neutral axis, and Hooke's Law is assumed to be valid for both steel and concrete. The tensile strength of the concrete is taken as equal to zero.

Due to the imposed deformations created by the prestressing, the total strains are not proportional to the distance from the neutral axis. Similar imposed deformations are created by the shrinkage of the concrete.

As long as the section remains uncracked, stress contributions from prestress, dead and live load, etc. may be calculated separately. Resulting stresses due to any load combination can then be calculated by superposition.

After crack formation, such superposition of stress contributions is no longer correct because the location of the neutral axis is not the same for the individual load contributions. Consequently, the internal forces must be calculated for the relevant load combinations. The section can then be analysed for these internal forces.

Whereas, for fully prestressed concrete, this complication only affects the analysis at the ultimate limit state, it must, for partially prestressed concrete, be taken into account also under service loading.

For structures with unbonded tendons, changes in strain cannot generally be assumed to be proportional to the distance from the neutral axis. The difference between the effects of unbonded and bonded tendons is more pronounced after crack formation than before.

Consequently, for fully prestressed structures this difference mainly influences the ultimate limit state. For partially prestressed structures, on the other hand, the difference may be of importance also under service loading.

Unless otherwise stated, the following discussion is confined to structures with bonded tendons.

3.7 NEUTRALIZATION

For a cross-section in a prestressed concrete beam or frame the concrete stresses at all points of the cross-section may be eliminated by adding a suitable, fictitious internal force, P_f. This situation is called neutralization. The corresponding stresses and strains in the steel are referred to as the neutralized stresses and strains and the corresponding value of the resultant of the stresses in the tendons is referred to as the neutralized prestressing force, P_n. If the effect of shrinkage is neglected, P_n will be the resultant of all stresses in the cross-section after neutralization. If the concrete stresses due to prestressing only, in a cross-section of a statically determinate structure, are neutralized, P_f and P_n will consequently be identical in magnitude, location and inclination.

The effect of shrinkage is discussed in section 4.3.

For the structural analysis of partially prestressed concrete it is convenient to adopt the neutralization as a basic reference level.

3.8 RELATIVE PRESTRESS

A symmetrical cross-section of a prestressed concrete beam with the prestressing steel located at one common level is shown in Fig. 2. At a certain bending moment, M_{pn}, the neutral axis will intersect the centroid of the prestressing steel, as illustrated in Fig. 2. The distribution of compressive concrete stresses above the neutral axis is assumed to be plane. Below the neutral axis the concrete stresses are neglected. The resultant N of the stresses in the concrete and in the non-prestressed steel passes through the point R of the cross-section. The lever arm of the neutralized prestressing force P_n with respect to R is denoted by z_p. The moment of P_n with respect to R is

$$\boxed{M_{pn} = P_n z_p}$$

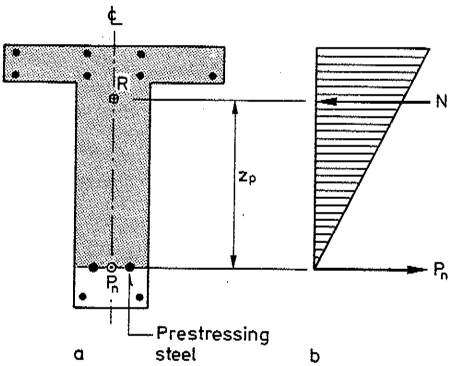


Fig. 2

The bending moment of the section under service loading is denoted by M. For statically indeterminate structures M includes the internal prestressing force. It is proposed here to define the relative prestress of a cross-section as

$$\boxed{\chi = \frac{M_{pn}}{M}}$$

If a cross-section with $\chi = 1$ is subjected to the service-load bending moment M, the combined effect of the load and prestress will correspond to a normal compressive force of the magnitude P_n passing through the point R in Fig. 2, and the neutral axis passes through the centroid of the prestressing steel.

For a cross-section without prestress χ is equal zero.

For prestressed sections the relative prestress χ is defined only if the service load has been determined.

Even fully prestressed sections may crack if the bending moment exceeds the design value. Consequently, a definition of full prestress must be tied up with the design value of the bending moment.

Plain concrete, reinforced concrete without prestressing, and fully prestressed concrete can be characterized as boundary cases of the general group: partial prestressed structures with mixed reinforcement. In the following, all types of structures will be considered as belonging to one continuous spectrum: Concrete structures. The relative prestress χ is believed to be a suitable parameter for indicating the location of a cross-section of a prestressed member in this spectrum.

4. ANALYSIS OF CRACKED CROSS-SECTION UNDER SERVICE LOADING

4.1 SIMPLE BENDING, CONSTANT WIDTH OF COMPRESSION ZONE, SAME EFFECTIVE DEPTH OF BOTH TYPES OF REINFORCEMENT

A symmetrical cross-section with constant width, b, of the compression zone is shown in Fig. 3.

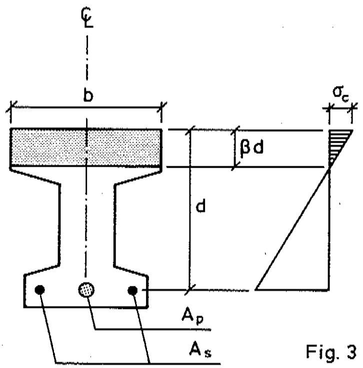


Fig. 3

The cross-sectional areas of the prestressed and non-prestressed reinforcements are denoted by A_p and A_s, respectively, and their strain moduli by E_p and E_s. Their effective depths, d_p and d_s, are - in the first instance - assumed to be of the same magnitude (d). Their modular ratios are

$$\alpha_{ep} = \frac{E_p}{E_c} \quad \text{and} \quad \alpha_{es} = \frac{E_s}{E_c}$$

where E_c denotes the strain modulus of the concrete. Their geometrical ratios of reinforcement are

$$\rho_p = \frac{A_p}{bd} \quad \text{and} \quad \rho_s = \frac{A_s}{bd}$$

The cross-section is assumed to be subjected to simple, symmetrical bending by a bending moment M.

For a rectangular cross-section the relative prestress according to Section 3.8 is

$$\chi = \frac{2 P_n d}{3 M}$$

This value of χ should also be adopted in the analysis of other cross-sections with constant width, b, of the compression zone. In other words, for such cross-sections χ is taken as the relative prestress of an equivalent rectangular cross-section with the same values of b, d and P_n.

The depth of the neutral axis is βd.

With the notation:

$$\Sigma \alpha \rho = \alpha_{ep} \rho_p + \alpha_{es} \rho_s$$

β is given by the equation

$$\beta^2 (3-\beta) \chi + 4(1-\beta) \Sigma \alpha \rho - 2\beta^2 = 0$$

This relation between β, χ and $\Sigma \alpha \rho$ is illustrated in Fig. 4.

Derivation of the equations is presented in [73- 1]*.

The maximum compressive concrete stress is

$$\sigma_c = \frac{M}{\mu_c \, bd^2} \quad \text{where} \quad \mu_c = \frac{1}{6} \beta (3-\beta)$$

*Numbers in square brackets refer to the Bibliography in Section 9.

The relation between μ_c, χ and $\Sigma \alpha \rho$ is illustrated in Fig. 5.

The tensile stresses in the prestressed and non-prestressed reinforcement are

$$\sigma_p = \sigma_{pn} + \Delta\sigma_p \quad \text{and} \quad \sigma_s = \sigma_{sn} + \Delta\sigma_s$$

where

$$\Delta\sigma_p = \frac{M}{\mu_s\, bd^2}\, \alpha_{ep}$$

$$\Delta\sigma_s = \frac{M}{\mu_s\, bd^2}\, \alpha_{es}$$

$$\mu_s = \frac{\beta^2 (3-\beta)}{6(1-\beta)}$$

σ_{pn} and σ_{sn} denote the corresponding neutralized stresses. The only possible (negative) contribution to σ_{sn} is due to shrinkage of the concrete. The relation between μ_s, χ and $\Sigma \alpha \rho$ is illustrated in Fig. 6.

The radius of curvature is denoted by r. Neglecting tension stiffening, the curvature of the beam is

$$\frac{1}{r} = \frac{M}{\beta \mu_c\, bd^3 E_c}$$

The relation between $\beta\mu_c$, χ and $\Sigma \alpha \rho$ is illustrated in Fig. 7.

For statically indeterminate structures the bending moment M includes the contribution corresponding to the internal prestressing force.

4. 2 COMPOUND BENDING

In the case of compound bending corresponding to a normal force N, which is taken as positive when compressive and which acts in the line of symmetry with the eccentricity e_p with respect to the reinforcement, this is equivalent to a pure bending moment

$$M = N\, e_p$$

plus an increase in the neutralized prestressing force from P_n to $P_n + N$. e_p is taken as positive when N is located on the same side of the reinforcement as the compression zone.

The diagrams in Figures 4-7 thus cover these cases too. The normal force may be due to either external load or internal prestressing forces.

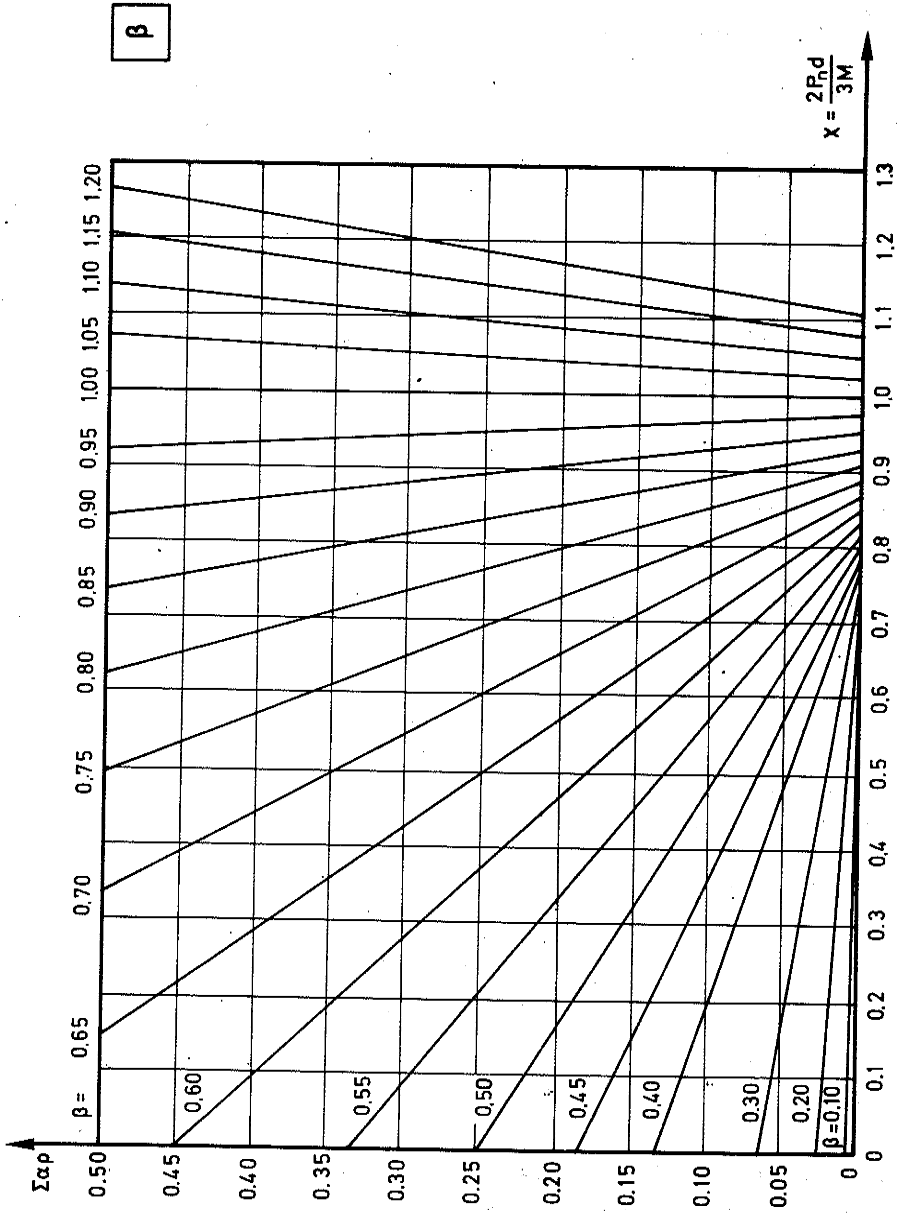
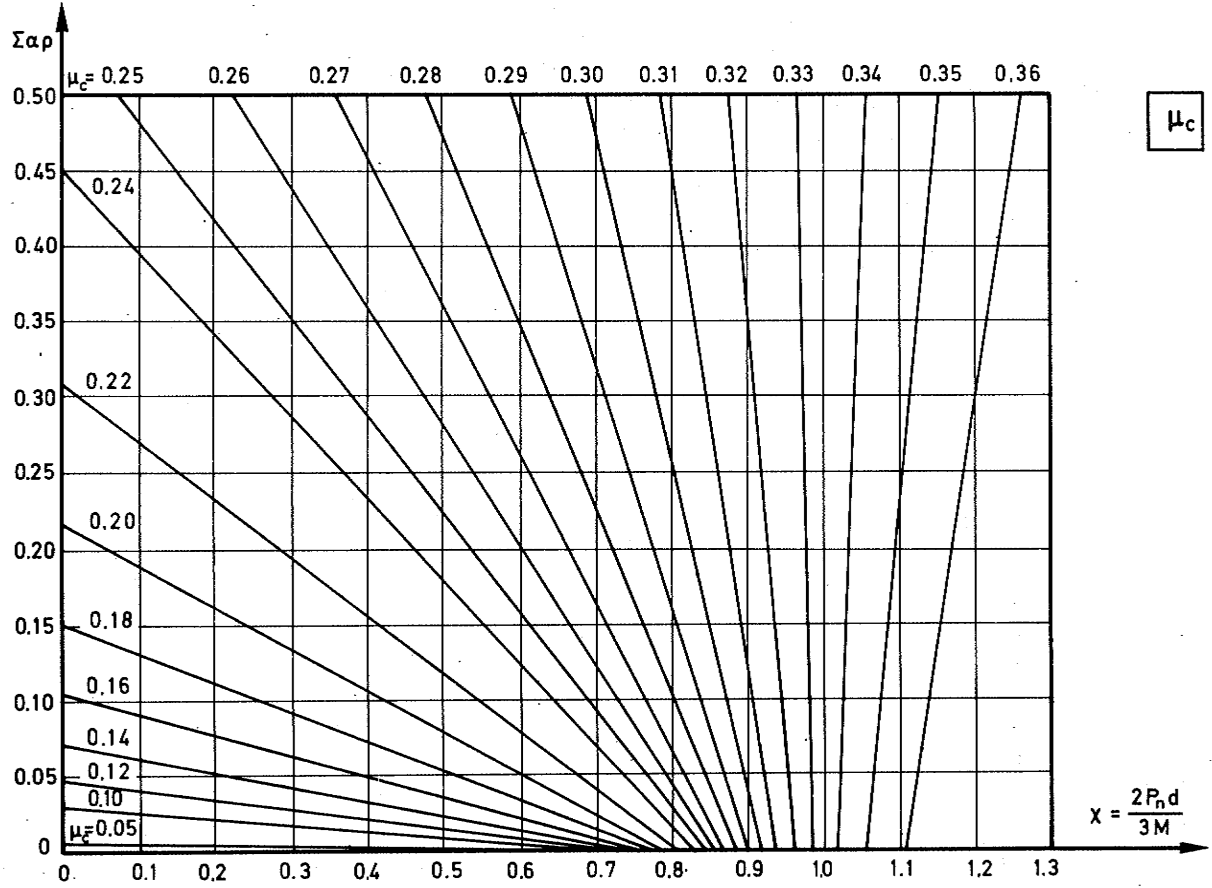

β
$X = \frac{2P_n d}{3M}$
Σαρ
β =
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
0.60
0.55
0.50
0.45
0.40
0.30
0.20
β = 0.10
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3

Fig. 4



Σαρ
μc= 0.25
0.26
0.27
0.28
0.29
0.30
0.31
0.32
0.33
0.34
0.35
0.36
μc
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
μc=0.05
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
X = 2Pnd / 3M

Fig. 5

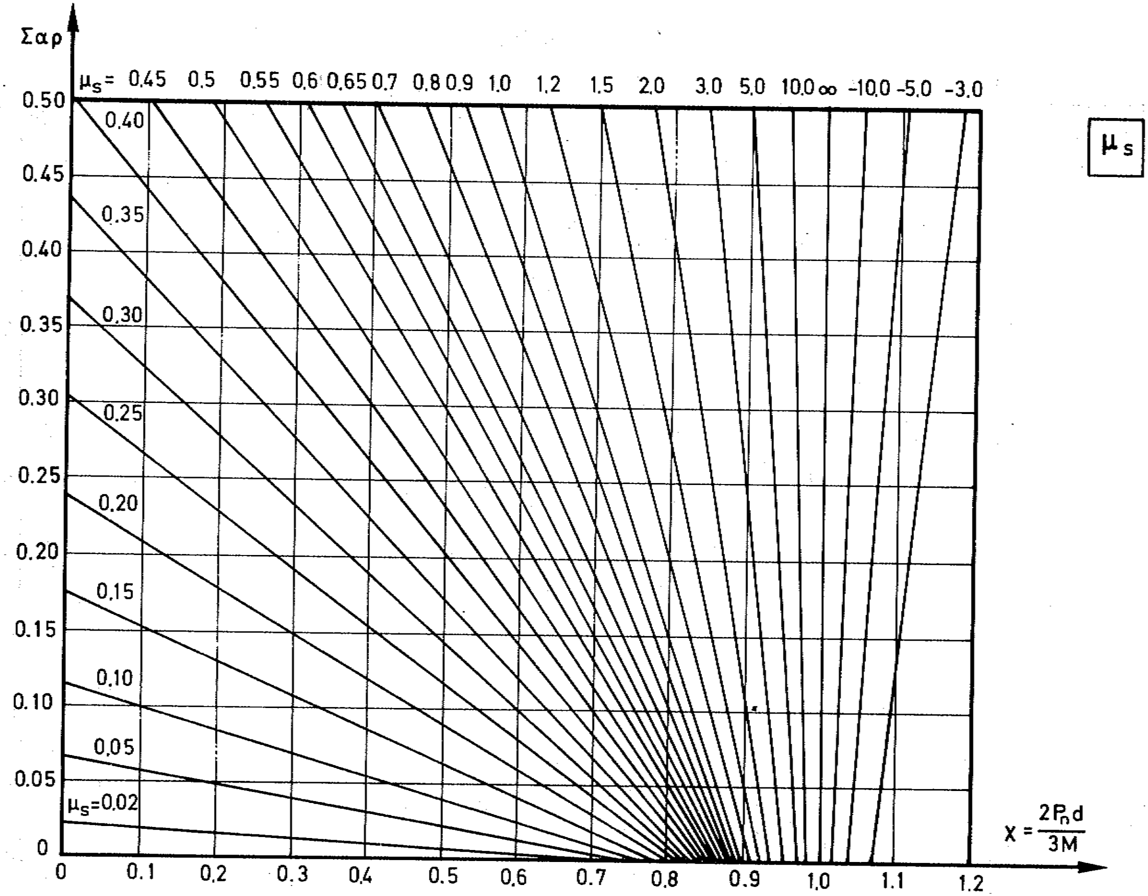
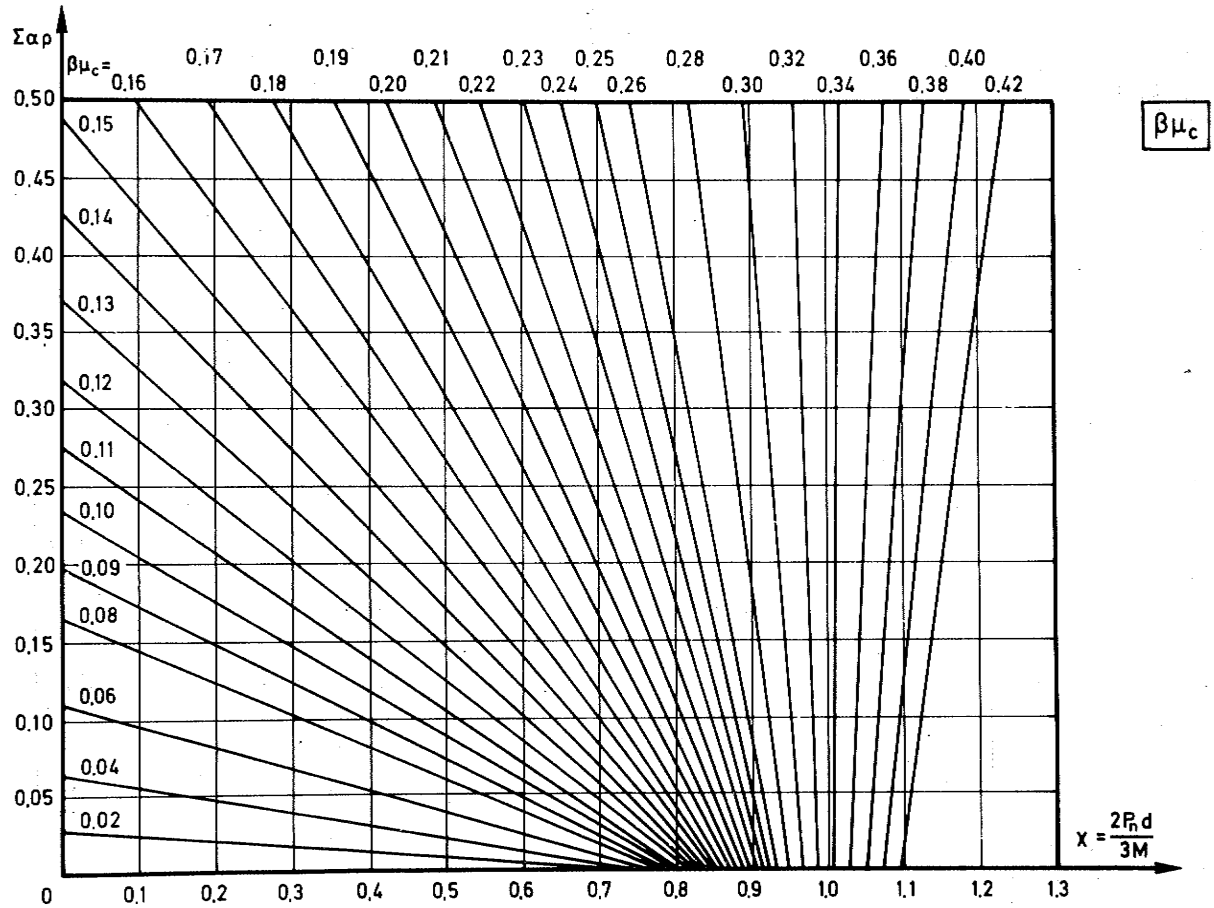

μs
Σαρ
μ_s= 0.45 0.5 0.55 0.6 0.65 0.7 0.8 0.9 1.0 1.2 1.5 2.0 3.0 5.0 10.0 ∞ -10.0 -5.0 -3.0
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
μ_s=0.02
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2
$X = \frac{2P_n d}{3M}$

Fig. 6



Σαρ
βμc=
0,16
0,17
0,18
0,19
0,20
0,21
0,22
0,23
0,24
0,25
0,26
0,28
0,30
0,32
0,34
0,36
0,38
0,40
0,42
βμc
0,15
0,14
0,13
0,12
0,11
0,10
0,09
0,08
0,06
0,04
0,02
0,50
0,45
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
1,1
1,2
1,3
X = 2Pnd/3M

Fig. 7

4.3 SHRINKAGE

The free (unstrained) shrinkage strain, ε_{cs}, of the concrete is defined here as the strain that would occur in the concrete if its stresses remained equal to zero and its temperature remained unchanged. ε_{cs} is taken as positive corresponding to a reduction in length.

The shrinkage can be taken into account by adding to the internal forces two fictitious forces:

$$N_{csp} = -\varepsilon_{cs} A_p E_p \qquad \text{and} \qquad N_{css} = -\varepsilon_{cs} A_s E_s$$

acting at the centroids of the prestressed and non-prestressed reinforcement, respectively. As normal forces are taken as positive when compressive, these forces are tensile forces.

In addition the shrinkage contributes to the neutralized stress in the non-prestressed reinforcement by the amount

$$\sigma_{sn} = -\varepsilon_{cs} E_s$$

i.e. a reduction in tensile stress. For prestressed reinforcement bonded to the concrete before the shrinkage takes place a corresponding contribution

$$\Delta \sigma_{pn} = -\varepsilon_{cs} E_p$$

should be added.

4.4 DIFFERENCE IN EFFECTIVE DEPTH OF PRESTRESSED AND NON-PRESTRESSED REINFORCEMENT.

In the case of a moderate difference between the effective depths, d_p of the prestressed and d_s of the non-prestressed reinforcement, a good approximation is obtained by the method suggested above by introducing for the effective depth, d, the effective depth of the centroid of the equivalent cross-sectional areas of reinforcement:

$$d = \frac{\alpha_{ep} A_p d_p + \alpha_{es} A_s d_s}{\alpha_{ep} A_p + \alpha_{es} A_s}$$

The moment of the neutralized prestressing force P_n with respect to this centroid should be added to the bending moment M from external load.

The stress contributions $\Delta\sigma_s$ and $\Delta\sigma_p$ resulting from this analysis are fictitious values corresponding to the level of the centroid of the equivalent reinforcements. By means of these values the real stress contributions corresponding to the individual layers of reinforcement can be calculated since these vary proportionally to the distance from the neutral axis.

The error introduced by this approximation corresponds to neglecting the moment of inertia of the equivalent reinforcements with respect to their own centroid in the calculation of the moment of inertia of the transformed cracked cross-section.

4.5 CREEP AND RELAXATION

Creep of concrete can be taken into account, approximately, by substituting for the strain modulus of concrete the so-called "effective modulus", i.e. a lower value for sustained loads than for instantaneous loads.

The relaxation of the prestessing steel may be correspondingly taken into account by substituting for the strain modulus of steel a lower value for sustained loads than for instantaneous loads.

Creep thus influences the values of the modular ratios α_{ep} and α_{es} and relaxation influences α_{ep}.

Even the very common combinations of sustained load and instantaneous load variations can be analysed according to this principle by means of a fictitious instantaneous neutralization. Details of the procedure were presented in [73- 1].

4.6 LOSSES OF TENSION

In conventional analysis the effective prestressing force is calculated by deducting losses due to shrinkage, elastic concrete strain, creep and relaxation, from the initial prestressing force. It is suggested here that the loss contributions from shrinkage, elastic concrete strain and creep be omitted and that these be taken into account, instead, in the analysis as explained in Sections 4.1, 4.3 and 4.5.

This should lead to more correct results.

As regards the loss due to relaxation, on the other hand, this should be taken into account in the usual way, and even with the full amount corresponding to the initial prestress. Reductions in this loss due to shrinkage, creep and elastic concrete strain can be taken into account in the analysis by a suitable reduction in the modular ratio, α_{ep}, of the prestressing steel. A more detailed explanation is given in [73- 1].

The effect of friction during tensioning of the tendons should be taken into account in the usual way.

4.7 GENERAL CASE

In the general case of biaxial bending and arbitrary cross-section, the analysis follows the same principles as explained in the previous sections. After neutralization, the stresses in the transformed, cracked cross-section, comprising concrete, prestressing steel and non-prestressed steel, should be calculated for a load equal to the resultant of P_n and the real internal force. The neutralized stresses should be added to these stresses.

5. ULTIMATE LIMIT STATE

5.1 BENDING AND COMPRESSION

The ultimate limit state can be checked according to the principles presented in the CEB-FIP Recommendations [70- 1]. For structures with mixed reinforcement the analysis should be based on the stress-strain curves for the two types of steel and on the neutralized strain of the prestressing steel.

The reinforcement is economically utilized in the ultimate limit state if the stresses in both types of reinforcement reach values in the vicinity of the design strength.

This can be achieved by proper choice of the neutralized prestress σ_{pn}. However, moderate variations in σ_{pn} have little effect on the ultimate load.

5.2 SHEAR

Further research is required to find out how prestressing influences the ultimate shear resistance. However, the influence, if any, appears to be moderate.

6. EFFECTS OF PRESTRESSING

As stated above, prestressing has only a moderate effect in the ultimate limit state, both in bending and in shear.

The main advantages of prestressing are the effects on the behaviour under service loading: reduction of crack widths, of deformations, and of stress variations. With the limitation of crack widths and deformations usually specified, the high strength of the tendons could not be utilized without prestressing.

The limitation of stress variation reduces the risk of fatigue. Although fatigue may result in a collapse, this usually takes place at such moderate stress levels that the analysis should be based on the assumptions specified for the service-load situation rather than on those adopted for the ultimate limit state check.

6.1 CRACK WIDTHS

According to the CEB-FIP Recommendations [70- 1] the crack widths for loads repeated more than 100 times are proportional to the amount by which the steel stress exceeds the neutralized stress, i.e. to the quantities $\Delta\sigma_s$ and $\Delta\sigma_p$ in Section 4.1 of the present paper.

The variation in these quantities with the bending moment M is illustrated in Fig. 8 for three cross-sections for which the ratios of prestressing steel (ϑ) are equal to 0.0, 0.5 and 1.0, respectively. Otherwise, the three cross-sections are identical and they have the same service loads and the same ultimate loads.

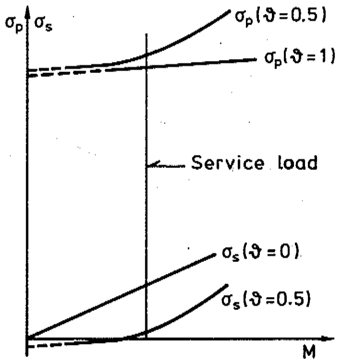


Fig. 8

The cross-section with $\vartheta = 1.0$ is fully prestressed, so no cracks should appear under service loading.

For the cross-section without prestressing ($\vartheta = 0$), a certain amount of cracking must be expected.

For the partially prestressed cross-section with mixed reinforcement ($\vartheta = 0.5$) cracks may occur under service loading, but the crack widths will only be a fraction of those in the non-prestressed beam.

In this connexion it should be borne in mind that the actual service load will only reach the design value in a few structures, and in still fewer, will this take place more than 100 times. Consequently, a partially prestressed beam will probably not crack in practice.

6.2 DEFORMATIONS

The deformations of a beam or frame structure can be calculated by double integration of the curvature, $\frac{1}{r}$.

The relation between the bending moment and the curvature is illustrated in Fig. 9 for three concrete beams with the same concrete cross-section and the same ultimate bending moment. The ratios of prestressing steel ϑ for the beams are equal to 0.0, 0.5 and 1.0, respectively.

In the case shown in Fig. 9 the beam with $\vartheta = 1$ is fully prestressed. For small values of M, it has a negative curvature (upward camber), which is just compensated for at service load. The relation between bending moment and curvature is linear.

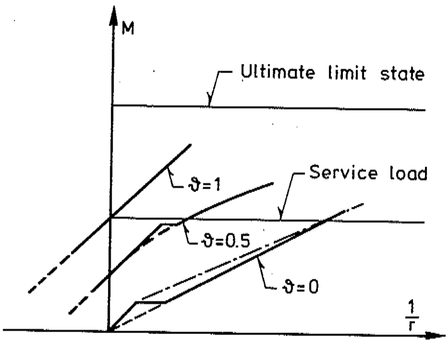


Fig. 9

The beam with $\vartheta = 0$ is a reinforced concrete beam without prestressing. When the beam is loaded for the first time, the theoretical relation between M and $\frac{1}{r}$ corresponds to the three straight lines drawn as full lines. For small values of M the cross-section is uncracked and $\frac{1}{r}$ is proportional to M. At the moment causing cracking there is an instantaneous increase in curvature. When M is increased above this moment, there is again proportionality between M and $\frac{1}{r}$, but now corresponding to the smaller bending rigidity of the cracked cross-section.

If the stress in the reinforcement does not exceed the elastic limit, the moment-curvature relation will follow the same straight line back to the origin if the beam is unloaded after cracking (indicated by a dotted line).

Due to the simplifications on which the analysis is based, the real moment-curvature relation deviates from this theoretical result. The crack pattern develops gradually and the uncracked parts between the cracks results in increased rigidity (tension stiffening). The main effect is that no instantaneous increase in curvature occurs for the bending moment causing cracking. A more realistic relationship is indicated with a dash-dotted line in Fig. 9.

For the partially prestressed beam with mixed reinforcement (ϑ = 0.5) the theoretical bending moment-curvature relationship is linear up to the moment causing cracking and curved for higher values of M. The full line in Fig. 9 corresponds to the first loading. For subsequent unloading and repeated loading, the dotted curve, corresponding to a cracked section, applies. As for the case ϑ = 0, these simplified results deviate somewhat from the physical reality.

6.3 FATIGUE

For structures subjected to load variations the safety against fatigue of the concrete, the prestressed, and the non-prestressed steel must be considered.

The stresses in the concrete and in the two types of reinforcement can be calculated for both the maximum and the minimum loading according to the principles described in Section 4, and the corresponding stress variations can be compared with the respective stress ranges causing fatigue.

The relation between the bending moment M and the steel stresses in a cracked cross-section is illustrated in Fig. 8. The corresponding concrete stresses are shown in Fig. 10.

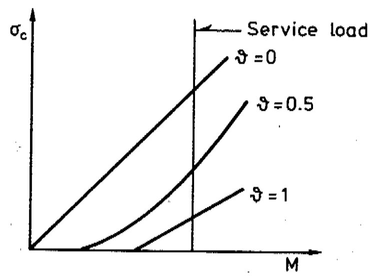


Fig. 10

The stress variations in both the prestressed and the non-prestressed reinforcement and in the concrete are reduced when the ratio of prestressing steel is increased.

For the concrete the increase in ratio of prestressing steel also reduces the service-load stress.

Due to all these effects, prestressing results in increased fatigue strength. However, partially prestressed structures may also be reliably designed against fatigue.

The fatigue strengths of the materials are often specified corresponding to $2 \cdot 10^6$ load repetitions. For most structures, the maximum variable load, which occurs $2 \cdot 10^6$ times, is only a small fraction of the variable load for which the structure is designed. This fact should be taken into account when designing structures against fatigue. It means that structures are much less sensitive to fatigue than they would be if the full design load were repeated $2 \cdot 10^6$ times.

7. ADVANTAGES OF PARTIAL PRESTRESSING WITH MIXED REINFORCEMENT AND FIELDS OF APPLICATION

Most of the advantages of prestressing also apply to partial prestressing. In this section some of the special advantages of partial prestressing and of the use of mixed reinforcement will be discussed.

As explained in Section 2, in the early days of prestressing the principle of prestressing was limitted to the idea of a fully prestressed concrete structure without cracks. Such a structure has obvious advantages. Frequently, however, the additional requirement that the structure be fully prestressed cannot be fulfilled without additional cost. In other words: partially prestressed structures may be more economical.

When evaluating the presumed main advantage of full prestress - that no cracks will develop in the concrete under service load - it should be realized that cracks also develop in most of the concrete structures that are considered to be fully prestressed.

Such cracks develop both in the anchorage zones and in other parts of the structure - mainly parallel to the tendons, which is likely to be the most dangerous orientation as regards protection against corrosion.

As explained in Section 3.8, partially prestressed structures with mixed reinforcement can be considered as the general group of concrete structures. Fully prestressed structures and reinforced concrete structures without prestressing represent special types of structures belonging to this group. The relative role of the prestressing steel in the ultimate limit state can be expressed by the ratio ϑ of prestressing steel defined in Section 3.3 and the relative prestress by the quantity X defined in Section 3.8.

In each specific case the optimum solution must be selected and, in many cases this will be a partially prestressed concrete structure with mixed reinforcement.

In the case of large spans, fully prestressed structures are likely to be the most economical solution. For short spans, reinforced concrete structures without prestressing often provide the most economical solution. For spans between these extremes reinforced concrete and full prestressing may compete, but partial prestressing with mixed reinforcement is likely to be the optimum solution.

The quantities of concrete and steel required in fully prestressed concrete structures are, in some cases, independent of the maximum and minimum loads and dependent only upon the load variation. This means that fully prestressed concrete is more likely to be competitive for structures with a moderate load variation relative to the minimum load. Correspondingly, fully prestressed concrete may be less economical when the load variation is predominant, for instance, if the bending moments vary between positive and negative extremes with approximately the same numerical values. This may apply, for instance, to masts and towers subjected to wind pressure in different directions.

If a structure has to be fully prestressed, this will often require the use of short tendons in order to eliminate local tensile stresses. This, for instance, is the case at intermediate supports for continuous beams, and at acute angles of skew slabs. Such short tendons are relatively expensive because the cost of anchorages, tensioning, grouting, etc. is approximately the same as for longer tendons.

By means of mixed reinforcement it is possible to utilize the advantages of both types of reinforcement. High-tensile prestressing steel should be utilized where it can be arranged as long tendons. Local zones, where the tensile stresses cannot be eliminated in this way, may more economically be provided with non-prestressed reinforcement.

When a prestressed beam carries its minimum load, the compressive concrete stresses must not exceed a certain limit. For this reason a considerable cross-sectional area of the bottom flange may be required in the case of full prestress. With partial prestress this area can be reduced. If the area is maintained or only partly reduced, the reduction in concrete stresses will result in reduced loss in tension due to creep of the concrete.

For structures that are fully prestressed corresponding to the maximum design load, the average (sustained) load will often be only a small fraction of the design load. This fact, in combination with creep of the concrete, may create problems due to upward deflections. Partial prestressing is an exellent means of controlling such deformations.

The permissible eccentricity of the prestressing force is limited in fully prestressed structures, particularly when the minimum load is small. The effective depth of the tendons is consequently smaller than the effective depth that can be obtained for non-prestressed reinforcement. For this reason partial prestressing with mixed reinforcement may offer advantages in the ultimate limit state.

The problem of the limited eccentricity of the tendons in fully prestressed structures can, in some cases, be overcome by tensioning in stages so that the final tensioning is delayed until most of the permanent load has been applied. However, this is a complication that can be avoided by partial prestressing and mixed reinforcement.

Shrinkage of the concrete may create harmful cracks before the tendons are tensioned. This risk is more pronounced in fully prestressed structures than in partially prestressed structures with mixed reinforcement because the larger ratio of non-prestressed reinforcement in the latter type of structure reduces the crack widths.

8. SUMMARY

Partially prestressed concrete structures with mixed reinforcement, i.e. a combination of prestressed and non-prestressed reinforcement, frequently offer an advantageous alternative to fully prestressed structures.

It is suggested that partially prestressed structures be characterized by the relative prestress rather than by the conventional degree of prestressing, which only depends upon the ratio of prestressing steel.

The effect of prestressing of statically indeterminate structures is explained as the combined effect of the internal prestressing forces and the residual stresses.

The analysis of cracked cross-sections under service loading is complicated by the fact that superposition of stress contributions from the various load contributions cannot be applied. This complication is overcome by the principle of neutralization of concrete stresses.

Principles, formulae and diagrams are given for the analysis of cracked cross-sections under service loading, including the effects of shrinkage, creep, relaxation and elastic strain on the losses of prestress.

In the ultimate limit state prestressing has little effect either on the resistance against bending and compression or on the shear capacity.

Under service loading - on the other hand - prestressing has an important effect on crack widths, deformations and resistance towards fatigue failure.

A number of advantages of partial prestressing with mixed reinforcement are listed.

The optimum solution - fully prestressed, partially prestressed, or reinforced concrete without prestressing - depends on a number of criteria such as span, type of loading and requirements with regard to crack widths, deformation and fatigue resistance.

Partial prestressing is likely to be the most economical solution for medium-span structures.

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