AFDELINGEN FOR

BÆRENDE KONSTRUKTIONER

DANMARKS TEKNISKE HØJSKOLE

STRUCTURAL RESEARCH LABORATORY
TECHNICAL UNIVERSITY OF DENMARK

M. W. Bræstrup, M. P. Nielsen and Finn Bach

RATIONAL ANALYSIS AND DESIGN OF STIRRUPS IN REINFORCED CONCRETE BEAMS

RAPPORT NR. R 79 1976

RATIONAL ANALYSIS AND DESIGN OF STIRRUPS IN REINFORCED CONCRETE BEAMS

by

M.W. Bræstrup, Lic.techn.
M.P. Nielsen, Professor, Dr.techn.
Finn Bach, Lic.techn.

Structural Research Laboratory
Technical University of Denmark

ABSTRACT

The shear strength of beams is analysed by the truss analogy with variable strut inclination. The web crushing criterion is derived as a solution satisfying equilibrium. If the materials are assumed to be perfectly plastic, the web crushing criterion is also an upper bound, corresponding to a failure mechanism with vertical deformations only. The solution is compared with experimental evidence and with the design rules of building codes, particularly the CEB Model Code.

TABLE OF CONTENTS

INTRODUCTION

The purpose of a beam is to transfer a load from its point of application to the support. This transfer causes diagonal tension cracks in the concrete, and unless the load is close to the support (compared with the beam depth) this means that the load will rest on the longitudinal reinforcement. If no countermeasures are taken, the reinforcing bars will be torn out of the concrete, and we get the type of failure shown on Fig.1. A diagonal crack runs from the load to the reinforcement and then splits the beam along the reinforcing bars. This diagonal tension failure should be avoided for two reasons. Firstly, it may occur at a load which is considerably lower than the flexural capacity of the beam. Secondly, it is a sudden failure which may cause disastrous collapse.

Diagonal tension failure may be prevented if the longitudinal bars are supported by an additional web reinforcement. Usually this shear reinforcement consists of closed stirrups encasing the longitudinal reinforcement and bent around the top bars or otherwise anchored in the compression zone.

Since the turn of the century, the action of the web reinforcement has been studied by hundreds of shear tests and dozens of theoretical investigations. Most of the latter are based upon the truss analogy, introduced by RITTER in 1899 and developed by MÖRSCH (cf. the historical study by HOGNESTAD [1]). A very important parameter of the analogy is the strut inclination, and the majority of present day building codes are based upon the truss model with 45° struts.

It has long been known that this inclination is not the one observed at shear failure of beams with web reinforcement, cf. CHAMBAUD [15], KUPFER [14]. A more realistic strut inclination may be determined by plastic analysis. In 1964, NIELSEN [2] considered reinforced concrete members in a state of plane stress. With the use of a lower bound method, formulas were derived for the stresses in reinforcement and concrete. Applying the theory to beams in shear, NIELSEN [3] in 1967 determined the strut inclination corresponding to minimum volume of reinforcement (longitudinal plus web).

The formulas are valid when both types of reinforcement are yielding without crushing of the concrete. In a subsequent discussion, NIELSEN [4] gave the strut inclination when the shear resistance is determined by the compressive strength of the concrete. The authors have recently realized that a similar equation had been proposed twelve years earlier by CHAMBAUD [13].

The ultimate load may also be determined by considering the mechanism of beams failing in shear. Most attempts in this direction have been based upon shear compression failure, where the beam end is rotating about a hinge in the compression zone (cf. the review in reference [5]). In 1975, NIELSEN & BRÆSTRUP [5] considered a pure shearing mechanism, without any rotation of the beam end. It was found that the corresponding ultimate load coincided with NIELSEN's lower bound corresponding to web concrete failure. This formula for the shear strength is termed the web crushing criterion.

The web crushing criterion has been compared with existing test results (reference [5]) and with shear tests carried out in Copenhagen (BRÆSTRUP et al.[6], BACH et al.[12]), and the agreement is found to be reasonable. Furthermore, the theory gives a rational explanation of the phenomena observed at shear failure, which is applicable not only to beam shear, but also to shear in joints and brackets, punching shear of slabs, torsion of beams, etc.

The purpose of the present paper is to give a review of the theory and its application for the design of stirrups in reinforced concrete beams.

NOTATIONS

A_s : Cross-sectional steel area of one stirrup

a : Length of shear span

b : Web width of beam

C : Compressive stringer force

c : Stirrup spacing along beam axis

d : Effective depth of beam

f_c : Compressive concrete cylinder strength

f_t : Uniaxial tensile concrete strength

f_y : Stirrup steel yield stress

h : Shear depth of beam

M : Applied bending moment

T : Tensile stringer force

T_y : Tensile stringer yield force

V : Applied shear force

z : Internal moment lever arm

α : Inclination of stirrups

β : Inclination of yield line

β_F : Yield line inclination corresponding to lowest upper bound

$\varepsilon_1, \varepsilon_2$: Principal strain rates

θ : Inclination of diagonal compression (concrete struts)

θ_F : Strut inclination corresponding to greatest lower bound

θ_{min}: Lower limit for allowable strut inclination

ν : Web effectiveness factor

ρ : Geometrical ratio of shear reinforcement

ρ_1 : Reinforcement ratio making beam overreinforced in shear

σ_a : Tensile stress in stirrup steel

σ_b : Compressive stress in web concrete

σ_1, σ_2: Principal stresses in concrete

Φ : Mechanical degree of longitudinal reinforcement

THE TRUSS MODEL

A simple way of visualizing the effect of the web reinforcement is by regarding the beam as a plane truss. The longitudinal bars and the stirrups (vertical or inclined) constitute the tension members. The compression members are formed by the concrete in the top chord and the web. The web width is termed b and the inclination of the stirrups is α. We introduce the geometrical ratio of shear reinforcement as:

$$\rho = \frac{A_s}{bc \sin\alpha} ,$$

where A_s is the cross-sectional steel area per stirrup and c is the stirrup spacing along the beam axis.

The truss analogy is given a precise formulation through the assumptions:

(a) The reinforcing bars are unable to resist lateral forces. The steel stress in the stirrups is σ_a. The compression zone and the longitudinal reinforcement act as stringers with a compressive force C and a tensile force T, respectively.

(b) The action of the stirrups is described by an equivalent stirrup stress $\rho\sigma_a$ per unit area perpendicular to the stirrup direction.

(c) The concrete of the web is in a state of uniaxial compression, the compressive stress σ_b being inclined at the angle θ to the beam axis.

Assumption (a) expresses that we neglect dowel action of the reinforcement and shear in the compression zone. The meaning of assumption (b) is that the spacing of the stirrups (longitudinally and transversely) is required to be sufficiently small to permit a description of their action as continuously distributed over a section perpendicular to the stirrups. Assumption (c) implies that the individual struts of the truss model are replaced by a diagonal compression field.

The mathematical model, taken to represent the beam, is shown on Fig.2. The beam depth h is defined as the distance between the compression and the tension stringer. For simplicity, we consider a part of a beam, the shear span a, with constant shear force.

A section of the beam is subjected to the shear force V and the bending moment M. Using the truss model, we find the equilibrium equations:

$$V = \sigma_b\, bh\cos\theta\,\sin\theta + \rho\sigma_a bh\cos\alpha\,\sin\alpha \tag{1}$$

$$M = hT - \frac{1}{2}\sigma_b\, bh^2\cos^2\theta + \frac{1}{2}\rho\sigma_a\, bh^2\cos^2\alpha \tag{2}$$

The condition that the stress be zero in a horizontal section leads to the relation:

$$\sigma_b \sin^2\theta = \rho\sigma_a \sin^2\alpha \tag{3}$$

Inserting (3) into (1) and (2), we find:

$$V = \rho\sigma_a bh\,\sin^2\alpha(\cot\theta + \cot\alpha) \tag{4}$$

and

$$M = hT - \frac{1}{2}\rho\sigma_a\, bh^2\,\sin^2\alpha(\cot^2\theta - \cot^2\alpha)$$

or

$$M = h[T - \frac{1}{2}V(\cot\theta - \cot\alpha)] \tag{5}$$

EQUILIBRIUM ANALYSIS

From the equilibrium equations, the load-carrying capacity of the beam may be derived if we introduce the material strength parameters. Thus we add the assumptions:

(d) The yield strength of the tensile stringer is $T = T_y$. The yield stress of the stirrup is $\sigma_a = f_y$.

(e) The crushing strength of the web concrete is $\sigma_b = \nu f_c$ where f_c is the cylinder strength and ν is a web effectiveness factor.

The beam is assumed not to be overreinforced in flexure, therefore the strength of the compression stringer is immaterial. With a fixed strut inclination θ, the shear strength is given by equation (4) with $\sigma_a = f_y$:

$$V = bh\rho f_y(\cot\theta + \cot\alpha)\sin^2\alpha \qquad (7)$$

Equation (7) is valid as long as the concrete strength of the web is not exceeded. By equation (3), this requires

$$\rho\sigma_a \leq \frac{\sin^2\theta}{\sin^2\alpha}\nu f_c$$

Inserting into equation (7), we find the strength limit imposed by the web concrete:

$$V \leq bh\nu f_c(\cot\theta + \cot\alpha)\sin^2\theta \qquad (8)$$

There is no reason to believe, however, that the strut inclination should remain constant. A generally accepted principle of mechanics states that the internal forces of a structure accomodate themselves to carry the maximum load. In the theory of plasticity, this principle is formalized as the lower bound theorem. From equation (7), we note that the flatter the concrete compression, the higher the shear force. Thus, if the ductility of the beam is sufficient, the web stresses will be distributed in such a way that the strut inclination decreases with increasing load. This effect is indeed observed during beam tests (cf. reference [6]). However, equation (3) imposes a lower limit on the strut inclination, i.e. an upper limit on the shear resistance. Eliminating θ between equation (3) and (4), we get:

$$V = bh\sqrt{\rho\sigma_a \sin^2\alpha(\sigma_b - \rho\sigma_a \sin^2\alpha)} + bh\rho\sigma_a \cos\alpha \sin\alpha \qquad (9)$$

By equation (9), V is an increasing function of σ_b, hence the maximum shear load is obtained at crushing of the concrete, $\sigma_b = \nu f_c$. Also, V is an increasing function of $\rho\sigma_a$, as long as

$$\rho\sigma_a \leq \frac{1}{2}\frac{1+\cos\alpha}{\sin^2\alpha}\nu f_c = \rho_1\sigma_a \qquad (10)$$

hence the maximum shear load is obtained with yielding of the stirrups, $\rho\sigma_a = \rho f_y$. Inserting into equation (9), we find the shear resistance as a function of the material strength parameters:

$$V = bh\sqrt{\rho f_y \sin^2\alpha(\nu f_c - \rho f_y \sin^2\alpha)} - bh\rho f_y \cos\alpha \sin\alpha \qquad (11a)$$

$$\text{valid for } \rho f_y \leq \rho_1\sigma_a$$

For $\rho f_y > \rho_1\sigma_a$, the maximum shear load is obtained with $\rho\sigma_a = \rho_1\sigma_a$, i.e. the stirrups do not yield at failure of the concrete. By equation (9), the shear strength is then:

$$V = \tfrac{1}{2}bh\nu f_c \cot\tfrac{\alpha}{2} \qquad (11b)$$

$$\text{valid for } \rho f_y \geq \rho_1\sigma_a$$

Equations (11) constitute the web crushing criterion. It gives the maximum shear force that can be carried by a particular concrete section. With a given shear reinforcement strength ρf_y, the optimal strut inclination is the one corresponding to failure of the web concrete. This value, $\theta = \theta_F$, is found from equation (3) with $\sigma_b = \nu f_c$ and $\rho\sigma_a = \rho f_y$ for $\rho f_y \leq \rho_1\sigma_a$ and $\rho\sigma_a = \rho_1\sigma_a$ for $\rho f_y \geq \rho_1\sigma_a$. Thus we get:

$$\cot\theta_F = \sqrt{\frac{\nu f_c}{\rho f_y \sin^2\alpha} - 1} \tag{12a}$$

valid for $\rho f_y \leq \rho_1\sigma_a$

and

$$\cot\theta_F = \tan\frac{\alpha}{2}$$

valid for $\rho f_y \geq \rho_1\sigma_a$ (12b)

If the beam is to achieve the maximum shear resistance given by the web crushing criterion, then it is a necessary condition that the tension stringer be sufficiently strong. By equation (5), this requires:

$$T_y \geq \frac{M}{h} + \frac{1}{2}V(\cot\theta - \cot\alpha) \tag{13}$$

Thus the tension stringer must be designed for a force which is greater than the pure bending term M/h. In particular, we note that a stringer force must be anchored at a simple support, where $M = 0$. Equation (13) does not apply at the maximum moment, because the diagonal compression field, used in deriving equation (5), is not valid (except possibly for indirect loading). At point loads and supports, the stress distribution must be modified, cf. NIELSEN [3] or NIELSEN & BRÆSTRUP [5]. If the shear reinforcement is very weak, the diagonal compression field degenerates to a single strut running from the load to the support. This case shall not be considered here (see the section below).

FAILURE MECHANISM

The upper bound method of the theory of plasticity may be used to determine an estimate of the ultimate load. However, in order to carry out a rigorous upper bound analysis, we must assume plastic properties of the materials. Thus we introduce the additional assumptions:

f) The stringers and the stirrups are rigid, perfectly plastic. The yield strengths are given by assumption d).

g) The web concrete is rigid, perfectly plastic with the square yield condition for plane stress and the associated flow rule. The tensile strength is zero and the compressive strength is νf_c .

These assumptions mean that the elastic deformations are neglected in the analysis. The yield locus for concrete in plane stress is shown on Fig.3. It is identical to the modified Coulomb failure criterion with a zero tensile cut-off. σ_1 and σ_2 are the principal stresses and the concrete is unable to resist stress combinations outside the square locus. The associated flow rule means that when the stress point is on the yield locus, then the ratio between the possible strain rates ε_1 and ε_2 is such that the vector $(\varepsilon_1,\varepsilon_2)$ is an outwards directed normal to the locus at the stress point. At the corners, the vector $(\varepsilon_1,\varepsilon_2)$ is situated between the adjacent normals.

A possible shear failure mechanism is shown on Fig.4. The deformations are taking place in yield lines at the inclination β , forming a parallelogram-shaped deformation zone. For comparison, Fig.5 shows a photograph of a test beam after failure. Note the absence of any rotation of the beam end, and the tensile cracks in the flange near the support which indicate that a yield zone has been formed.

Using assumptions f) and g) , the rate of internal work dissipated in the failure mechanism is calculated. An upper bound for the ultimate shear force is found be equating the rate of internal work to the rate of external work done by the load. The lowest upper bound is determined by minimizing with respect to the yield line inclination β . As shown by NIELSEN & BRÆSTRUP [5], the result is identical with the web crushing criterion, equations (11). Since this solution is also a lower bound, it is in fact the complete solution corresponding to the assumptions made.

In the failure mechanism giving the lowest upper bound, the inclination $\beta = \beta_F$ of the yield lines is:

$$\beta_F = 2\theta_F \tag{13}$$

where θ_F is the strut inclination given by equations (12), corresponding to failure of the web concrete. The fact that strut inclination is different from yield line inclination, means that shear stresses are transferred in the yield lines (possibly by aggregate interlock). The situation is similar to a compressed concrete cylinder failing along an inclined plane.

When the shear reinforcement is very weak, the deformation zone degenerates into a single yield line running from the load to the support. Thus we have

$$\cot\beta_F = a/h \tag{14}$$

where a is the length of the shear span, measured between the edges of the support and the load platens. A treatment of this case is outside the scope of the present paper (see reference [7]). Let it just be mentioned that for beams without shear reinforcement, coincident upper and lower bounds may be found, viz.:

$$V = \frac{1}{2}bh\nu f_c \left(\sqrt{(\frac{a}{h})^2 + \frac{4\Phi(\nu-\Phi)}{\nu^2}} - \frac{a}{h} \right) \tag{15a}$$

$$\text{valid for} \quad \Phi \leq \frac{1}{2}\nu$$

and

$$V = \frac{1}{2}bh\nu f_c\left(\sqrt{\left(\frac{a}{h}\right)^2 + 1} - \frac{a}{h}\right) \qquad (15b)$$

valid for $\Phi \geq \frac{1}{2}\nu$

Here Φ is the degree of longitudinal reinforcement, defined as

$$\Phi = \frac{T_y}{bh\,f_c}$$

Equation (15b) was given by NIELSEN & BRAESTRUP [5].

APPLICATION OF THE THEORY

The formulas for the shear resistance of reinforced concrete beams are visualized on Fig.6 in the case of vertical stirrup ($\alpha = 90^{\circ}$). The non-dimensional shear strength V/bhf_c is shown as a function of the mechanical degree of shear reinforcement $\rho f_y/f_c$. Equation (7) corresponds to a straight line with the inclination $\cot\theta$. The web crushing criterion, equations (11), is represented by a quarter-circle with diameter ν and centre at $(\nu/2,\ 0)$, plus the horizontal tangent.

Suppose we have chosen a fixed strut inclination θ . The shear strength as a function of the stirrup reinforcement is then given by equation (7), until it reaches the limit determined by equation (8) and represented by the circle on Fig.6. Then the shear capacity can be increased no further, unless greater dimensions or stronger concrete are prescribed.

It is more reasonable to assume that the strut inclination varies with the shear load. The most economical inclination is $\theta = \theta_F$, corresponding to the web crushing criterion. It is determined by equation (12a) , where the necessary stirrup reinforcement ρf_y is found from equation (11a), inserting the applied shear force V . This shear load must be inferior to the upper limit given by equation (11b).

For weak shear reinforcement, the web crushing inclination θ_F is very small. Therefore the design may be unfeasible, due to the increase in tensile stringer force, as given by equation (13). Also, the stress distribution at failure will be very different from the one at service load, leading to unacceptable requirements to concrete ductility. For these reasons, it is advisable to impose a minimum strut inclination $\theta = \theta_{min} < 45^{\circ}$. This means that equation (7) with $\theta = \theta_{min}$ determines the shear strength until the limit set by equation (8). Then the shear strength is given by the web crushing criterion, equation (11a), with $\theta = \theta_F$ until the limit given by equation (11b). From that point the shear strength cannot be increased by adding more stirrup reinforcement.

In order to use the web crushing criterion for design, it is necessary to assess the values of the shear depth h and the effectiveness factor ν by correlation with experimental evidence. Fig.7 shows some test results reported by LEONHARDT & WALTHER [8]. The series comprises 18 beams with vertical stirrups. In two of these, the main reinforcement was curtailed, and in three it was bent up. One beam had additional shear reinforcement in the form of horizontal bars. Of the remaining 12 beams, three have been omitted from the plot because they were reported to have failed in flexure. The non-dimensional shear strength is plotted against the shear reinforcement degree. As shear depth is used the internal moment lever arm z, calculated as the distance between the centroid of the main reinforcement and the centre of the compression flange. For comparison is shown the web crushing criterion with $\nu = 0.86$, which is the value giving closest fit by orthogonal regression. The coefficient of variation is 2.0%.

The web effectiveness factor ν depends on the concrete ductility and on the lay-out of the reinforcement. However, for reasonably designed beams, ν appears to be fairly constant. Fig.8 shows the results of 178 shear tests on beams with vertical stirrups. 72 tests have been carried out recently at the Structural Research Laboratory (references [6] and [12]), while the rest are reported in the literature (references [8], [13], and [17] - [28]. In cases where the cylinder strength is not given, f_c is taken as 80% of the cube strength. Flexural and bond failures are omitted (detailed documentation is available from the authors, who would also appreciate information about test series not included). The results are plotted as on Fig. 7, and the mean effectiveness factor is $\nu = 0.74$, the coefficient of variation being 6%.

COMPARISON WITH BUILDING CODES

Proposals for the use of the web crushing criterion for the design of stirrup reinforcement are given in reference [7] and shall not be repeated here. Instead, we shall compare the theoretical formulas, derived in the preceeding sections, with the design rules of building codes.

Fig.8 shows the shear capacity as calculated by the Danish Code of Practice, DS 411 [9],[10]. The code requires the use of the internal moment lever arm z as shear depth, and a strut inclination of $\theta = 45^{\circ}$. Thus the shear strength is given by equation (7) with $h = z$ and $\theta = 45^{\circ}$. As upper limit on the shear load is imposed the value $V = 0.25\ f_c\ bz$. Comparing with equation (11b), we see that this corresponds to an effectiveness factor $\nu = 0.50$. For 45° stirrups, the upper limit is $V = 0.45\ f_c\ bz$, corresponding to an effectiveness factor $\nu = 0.37$. In addition, the code allows a 'shear contribution from the concrete' of $V = 0.5\ f_t\ bz$, f_t being the uniaxial tensile strength. (In Fig.8, the actual strength parameters have been used. Of course, design values are to be inserted when the code is applied). This additional term is devoid of any theoretical justification when shear reinforcement is present. It seems mainly to be included to compensate for the unfavourable choice of strut inclination.

Nevertheless, it is obvious from Fig.8 that the code is very conservative, and that even with an effectiveness parameter as low as $\nu = 0.50$, the use of the web crushing criterion would lead to a substantial saving of stirrups for small and moderate degrees of shear reinforcement.

The latest code proposal from CEB [11] suggests a socalled 'refined method' for the design of shear reinforcement, using variable strut inclination. In clause 11.2.4.2 of the Model Code (equation [11.19]), we find equation (7) with a shear depth $h = 0.9\ d$, d being the effective depth of the beam. As a lower limit for the strut inclination in proposed $\cot\theta_{min} = 2.0$. The shear strength limit imposed by the web concrete is given by equation [11.17] of the Model Code, which corresponds to equation (8) with $\nu = 0.60$ and $h = d$. However, the applications of this equation is restricted by the requirement

$$V < 0.45\ f_c\ bd\ \sin 2\theta$$

The 'concrete term' which is given as $V = 0.6\ f_t\ bd$ for very small shear loads, is very reasonably phased out when any significant shear reinforcement is necessary. However, a rational estimate of the shear strength of beams without shear reinforcement should include the effects of the shear span ratio a/h and the longitudinal reinforcement degree Φ , as is the case with equations (15), given above.

The design of the main reinforcement requires a special note. According to equation [11.20] of the Model Code, the tensile stringer force is increased (with respect to the simple moment term) by the amount:

$$\Delta T_y = \frac{V^2 c}{2A_s\ f_y d} \tag{16}$$

using the notation of the present paper. Assuming $\sigma_a = f_y$ and $h = d$, the applied shear force is given by equation (4):

$$V = \rho f_y\ bd\ \sin^2\alpha(\cot\theta + \cot\alpha)$$

$$= \frac{A_s}{c} f_y\ d\ \sin\alpha(\cot\theta + \cot\alpha)$$

Inserting into equation (16), we find

$$\Delta T_y = \frac{1}{2} V\ \sin\alpha(\cot\theta + \cot\alpha)$$

On the other hand, equation (13) requires

$$\Delta T_y = \frac{1}{2} V(\cot\theta - \cot\alpha)$$

Thus we note that equation (16) is correct in the case of vertical stirrups ($\alpha = 90^{\circ}$) , but generally not when the stirrups are inclined.

CONCLUSION

In the preceding sections, we have shown that a rational analysis of the shear strength of reinforced concrete beams with stirrups may be based upon the truss analogy with variable strut inclination. The assumption of perfectly plastic properties of the materials leads to a solution, the web crushing criterion, which is both an upper and a lower bound. The web crushing criterion is found to agree reasonably well with experimental evidence, provided we introduce an empirical web effectiveness factor.

An important step towards the application of the web crushing criterion in practical design is taken by the CEB Model Code. It should be noted, however, that the formula for the increase of main reinforcement due to shear is incorrect in the case of inclined stirrups. The Model Code almost abolishes the socalled addition principle, i.e. the inclusion of a shear stress term proportional to the tensile concrete strength. Still wanting is a formula for the shear strength of beams with little or no stirrups, taking account of the shear span ratio and the amount of longitudinal reinforcement.

ACKNOWLEDGEMENTS

The present paper was written during the research fellowship (seniorstipendium) of one author (MWB) at the Structural Research Laboratory, Technical University of Denmark. The test series mentioned and part of the theoretical investigation reported were sponsored by the Danish Council for Scientific and Technical Research.

REFERENCES

[1] HOGNESTAD, E.:

What do we know about diagonal tension and web reinforcement in concrete? A historical study.
Urbana. University of Illinois Engineering Experiment Station. Circular Series No.64 (Bulletin Vol.49, No.52). 1952. pp.47.

[2] NIELSEN, M.P.:

Yield conditions for reinforced concrete shells in the membrane state.
Amsterdam. Proc. IASS Symposium on Non-classical Shell Problems, Warsaw 1963. (ed. Olszak & Sawczuk). 1964. pp. 1030-1040.

[3] NIELSEN, M.P.:

Om forskydningsarmering i jernbetonbjælker.
(On shear reinforcement in reinforced concrete beams.)
Bygningsstatiske Meddelelser. Vol.38, No.2. 1967.pp.33-58.

[4] NIELSEN, M.P.:

Discussion on [3].
Bygningsstatiske Meddelelser. Vol.40, No.1.1969. pp.55-63.

[5] NIELSEN, M.P. & BRAESTRUP, M.W.:

Plastic shear strength of reinforced concrete beams.
Bygningsstatiske Meddelelser. Vol.46, No.3. 1975.p.61-99.

[6] BRAESTRUP, M.W., NIELSEN, M.P., BACH, F. & JENSEN, B.C.:

Shear tests on reinforced concrete T-beams. Series T.
Copenhagen. Technical University of Denmark. Structural Research Laboratory. Report No.R72. 1976. pp.114.

[7] NIELSEN, M.P., BRAESTRUP, M.W., JENSEN, B.C. & BACH, F.:
Concrete plasticity. Beam shear - punching shear - shear in joints.
Copenhagen. Danish Society for Structural Science and Engineering. Special publication under preparation.

[8] LEONHARDT, F. & WALTHER, R.:

Schubversuche an Plattenbalken mit unterschiedlicher Schubbewehrung.
Berlin. Deutscher Ausschuss für Stahlbeton. Heft 156.1963. pp.84.

[9] Dansk Ingeniørforening:

Code of practice for the structural use of concrete.
Danish Standard DS 411.
Copenhagen. Teknisk Forlag. Normstyrelsens Publikationer, NP-116-T. 2.edition 1976. pp.63.

[10] Dansk Ingeniørforening:

Supplementary guide to code of practice for the structural use of concrete. Supplement to Danish Standard DS 411.
Copenhagen. Teknisk Forlag. Normstyrelsens Publikationer, NP-117-T. 2.edition 1976. pp.52.

[11] Comité Euro-International du Béton:

Code Modèle pour les structures en béton.
Bulletin d'Information No.117-F. December 1976. pp.301.

[12] BACH, F., NIELSEN, M.P. & BRAESTRUP, M.W.:

Shear tests on reinforced concrete T-beams. Series V, U, X, B, and S.
Copenhagen. Technical University of Denmark. Structural Research Laboratory. Report under preparation.

[13] CHAMBAUD, R.:

La rupture par flexion et par effort tranchant dans des poutre en béton armé.
Annales de l'Institut Technique du Bâtiment et des Traveaux Publics. No.110. Feb 1957. pp.167-206.

[14] KUPFER, H.:

Erweiterung der MÖRSCH'schen Fachwerkanalogie mit Hilfe des Prinzips vom Minimum der Formänderungsarbeit.
Comité Européen du Béton. Bulletin d'Information. No.40. Jan 1964. pp.44-57.

[15] CHAMBAUD, R.:

Le calcul à la rupture par flexion et par effort tranchant dans des pièces en béton armé.
Association International des Ponts et Charpentes.
V^{e} Congrès. Lisboa-Porto 1956. Rapport Final. 1957. pp. 551-555.

[16] TAYLOR, R.:

Some tests on reinforced concrete beams without shear reinforcement.
Magazine of Concrete Research. Vol.12, No.36. 1960. pp. 145-154.

[17] HAGBERG, Th.:

Forsøk med betongbjelker med spesielle lagerbetingelser.
Technical University of Trondheim. Institute for Concrete and Concrete Structures. Betongtekniske Publikasjoner Nr.8. 1967. pp.36-55.
(See also: LEONHARDT, F., WALTHER, R. and DILGER, W.: Schubversuche an indirekt gelagerten, einfeldringen und durchlaufenden Stahlbetonbalken. Deutscher Ausschuss für Stahlbeton. Heft 201. 1968. pp.69).

[18] LEONHARDT, F. & WALTHER, R.:

Schubversuche an einfeldringen Stahlbetonbalken mit und ohne Schubbewehrung.
Deutscher Ausschuss für Stahlbeton. Heft 151. 1962. pp.83.

[19] LEONHARDT, F. & WALTHER, R.:

Geschweisste Bewehrungsmatten als Bügelbewehrung. Schubversuche an Plattenbalken und Verankerungsversuche.
Die Bautechnik. Vol.42,No.10. 1965. pp.329-341.

[20] MALLING, V.:

Forskydningsforsøg med jernbetonbjælker med kraftig Bøjlearmering.
Aalborg. Danmarks Ingeniørakademi. Bygningsafdelingen. Ren og Anvendt Mekanik. Report 7202. 1972. pp.9.

[21] MOAYER, M. & REGAN, P.E.:

Shear strength of prestressed and reinforced concrete T-beams.
American Concrete Institute. Special Publication SP-42. 1974. pp.183-213.

[22] PETERSSON, T.:

Rektangulära och T-formade betongbalkars skjuvhållfasthet - en jämförelse.
Stockholm. Byggforskningen. Report 111. 1964. pp.140.

[23] RATHKJEN, A.:

Forsøg med bjælker med afkortet armering.
Aalborg. Danmarks Ingeniørakademi. Bygningsafdelingen. Ren og Anvendt Mekanik. Report 6901.1969. pp.14.

[24] REGAN, P.:

Shear in reinforced concrete. An experimental study.
London. Imperial College of Science and Technology. Department of Civil Engineering. A report to the construction Industry Research and Information Association. 1971. pp.203.

[25] SWAMY, R.N. & QURESHI, S.A.:

Shear behaviour of reinforced concrete T-beams with web reinforcement.
Institution of Civil Engineers. Proceedings Part 2, Research and Theory. Vol.57. Mar. 1974. pp.35-49.

[26] SØRENSEN, H.C.:

Forskydningsforsøg med 12 jernbetonbjælker med T-tværsnit.
Copenhagen. Technical University of Denmark. Structural Research Laboratory. Report R 20. 1971. pp.49.
(English translation: Shear tests on 12 reinforced concrete beams. Report R 60. 1974. pp.49).

[27] TAYLOR, R.:

Some shear tests on reinforced concrete beams with stirrups.
Magazine of Concrete Research. Vol.18, No.57. 1966. pp. 221-230.

[28] ÖZDEN, K.:

An experimental investigation on the shear strength of reinforced concrete beams.
Technical University of Istanbul, Faculty of Civil Engineering. 1967. pp.243.

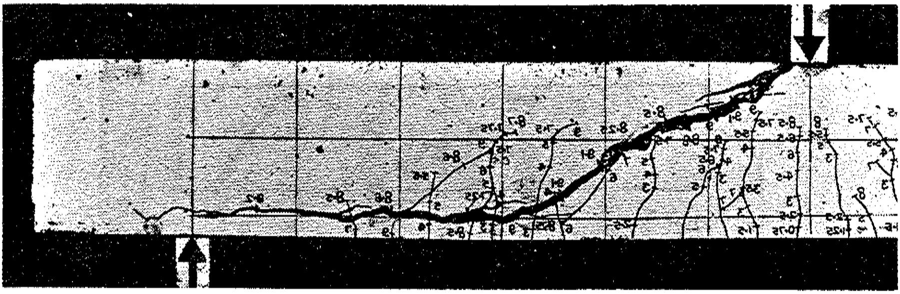

Figure 1: Diagonal tension failure of beam without web reinforcement (reproduced from TAYLOR [16]).

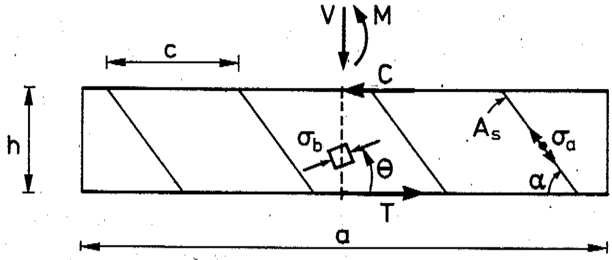

Figure 2: Truss model of reinforced concrete beam.

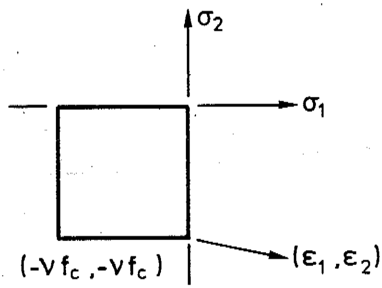

Figure 3: Yield locus for concrete in plane stress.

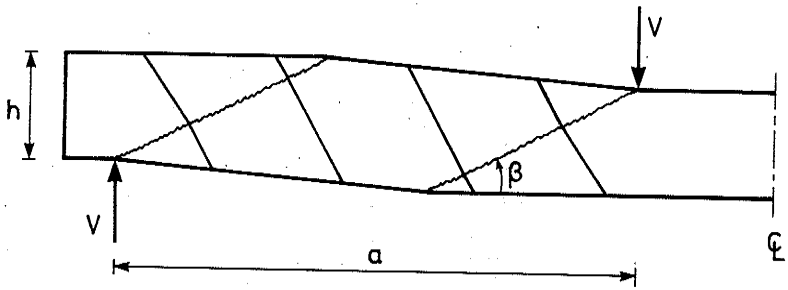

Figure 4: Shear failure mechanism for reinforced concrete beam.

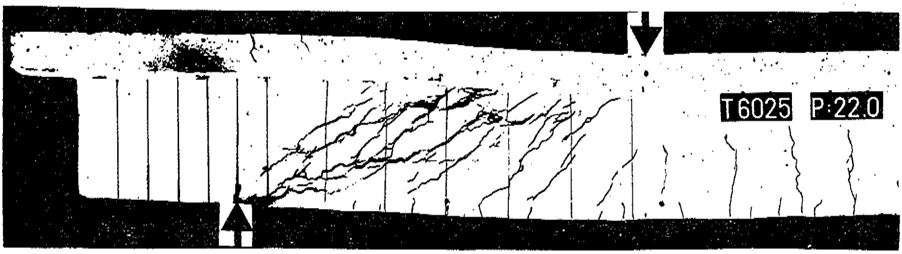

Figure 5: Shear failure of beam with web reinforcement (BRÆSTRUP et al. [6]).

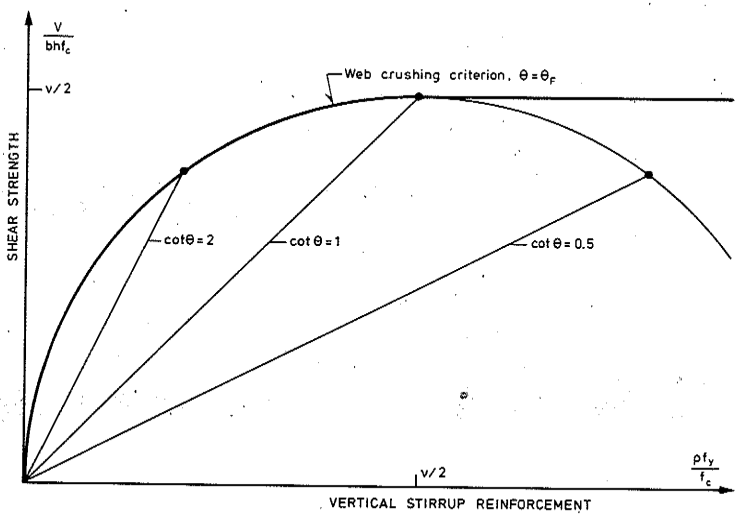

Figure 6: Shear resistance of reinforced concrete beams (vertical stirrups).

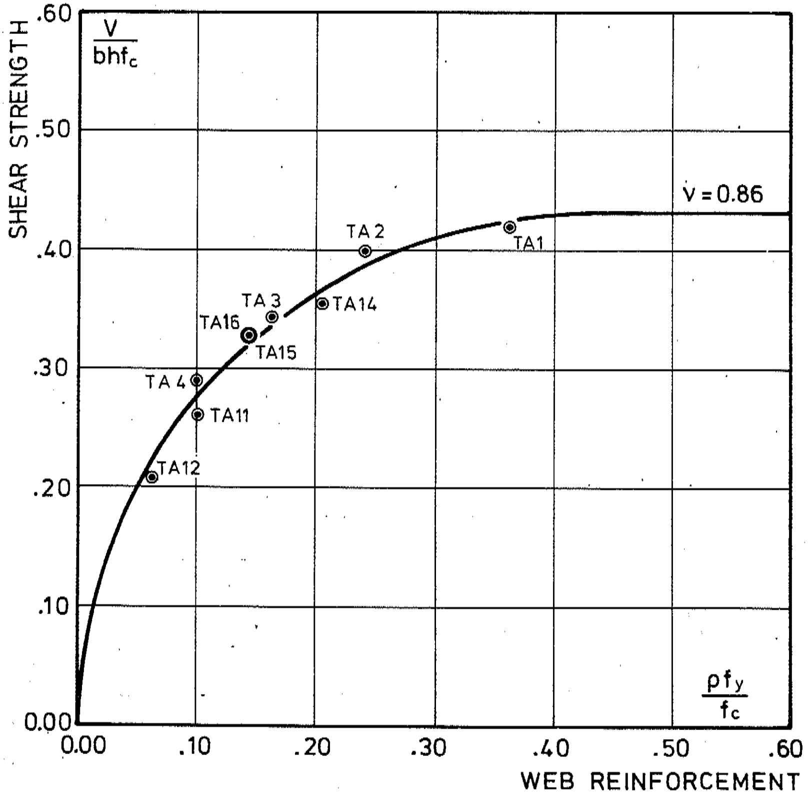

Figure 7: Web crushing criterion compared with test results (LEONHARDT & WALTHER [8]).

RESULTS OF 178 SHEAR TESTS

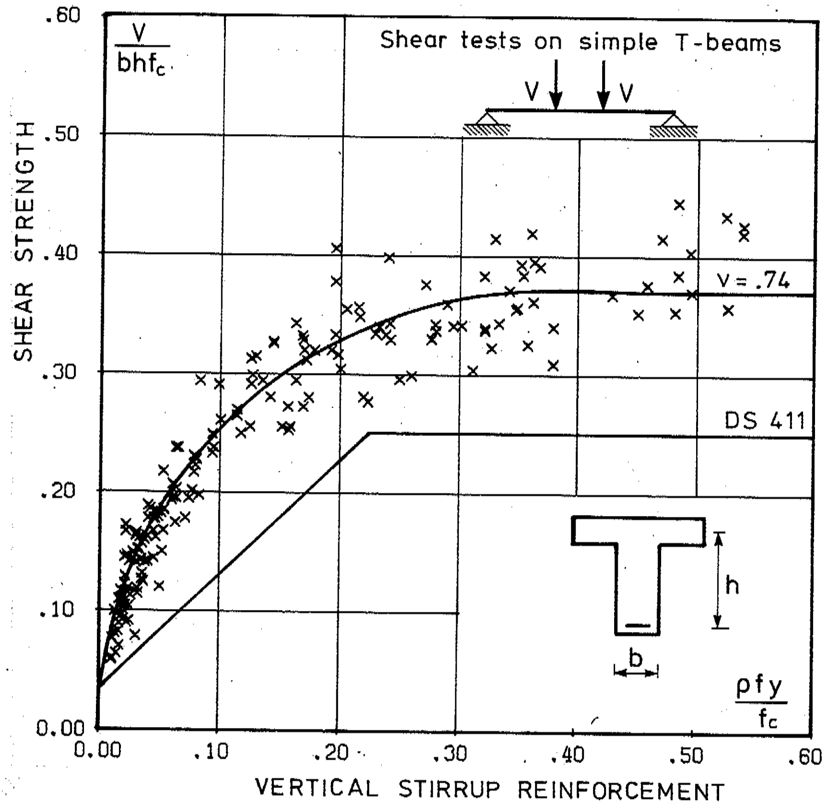

Figure 8: Results of shear tests compared with web crushing criterion and Danish building code.

AFDELINGEN FOR BÆRENDE KONSTRUKTIONER
DANMARKS TEKNISKE HØJSKOLE

Structural Research Laboratory
Technical University of Denmark, DK-2800 Lyngby

RAPPORTER (Reports)
(1974 -)

R 43. BORCHERSEN, EGIL: Moiré pattern deformation theory and optical filtering techniques. 1974.
R 44. BRØNDUM-NIELSEN,TROELS: Optimum design of reinforced concrete shells and slabs. 1974.
R 45. PEDERSEN, FLEMMING BLIGAARD: Dynamic properties of anti-vibration mountings. 1974.
R 46. PHILIPSEN,CLAUS: Interferensholografisk bestemmelse af legemers form og flytningsfelt. 1974.
R 47. LARSEN,H.J. og H.RIBERHOLT: Tværbæreevne af søm og dykkere i spån- og træfiberplader. 1974.
R 48. POULSEN,P.E.: The photo-elastic effect in three-dimensional states of stress. 1974.
R 49. NIELSEN,J.: Modellove for kornede medier med særligt henblik på silomodeller. 1974.
R 50. KRENK,STEEN: The problems of an inclined crack in an elastic strip. 1974.
R 51. BRØNDUM-NIELSEN,TROELS: Effect of prestress on the damping of concrete. Effect of grouting on the fatique strength of post-tensioned concrete beams. 1974.
R 52. EGERUP,ARNE RYDEN,H.J.LARSEN,H.RIBERHOLT and ERIK SØRENSEN: Papers presented at IUFRO-V, International Union of Forestry Research Organisation, Division V, Congress 1973. 1974.
R 53. HOLST,OLE: Automatic design of plane frames. 1974.
R 54. NIELSEN,SØREN: Svingninger i mastebarduner. 1974.
R 55. AGERSKOV,HENNING: Behaviour of connections using prestressed high strength bolt loaded in tension. 1974.
R 56. MØLLMANN,H.: Analysis of prestressed cable systems supported by elastic boundary structures. 1974.
R 57. NIELSEN,J. and V.ASKEGAARD: Scale errors in model tests on granular media with special reference to silo models. 1974.
R 58. SVENSSON,SVEN EILIF: Stability properties and mode interaction of continuous, conservative systems. 1974.
R 59. SIGBJÖRNSSON,RAGNAR: On the theory of structural vibrations due to natural wind. 1974.
R 60. SØRENSEN,HANS CHR.: Shear tests on 12 reinforced concrete T-beams. 1974.

R 61. NIELSEN,LEIF OTTO: Spændingshybride finite elementer til svingningsproblemer. 1975.
R 62. EGERUP,ARNE RYDEN: Theoretical and experimental determination of the stiffness and ultimate load of timber trusses. 1975.
R 63. LAURSEN,MARTIN: A curved beam equilibrium element applicable in standard finite element program systems. 1975.
R 64. BACH,FINN: Metoder til måling af egenspændinger. 1975.
R 65. BACH,FINN: En teoretisk og eksperimentel undersøgelse af den akustoelastiske metodes anvendelighed til egenspændingsmåling. 1975.
R 66. PEDERSEN,FLEMMING BLIGAARD: Measurement of the complex modulus of viscoelastic materials. 1975.
R 67. PEDERSEN,FLEMMING BLIGAARD: Svingningsforsøg med viskoelastisk dæmpede sandwichbjælker. 1975.
R 68. AGERSKOV,HENNING: Analysis of high strength bolted connections subject to prying. A simplified approach. 1975.
R 69. PEDERSEN,MAX ELGAARD: En 2.ordens tilnærmelse til de konstitutive ligninger for beton. 1976.
R 70. RIBERHOLT,HILMER and PETER CHR. NIELSEN: Timber under combined compression and bending stress. 1976.
R 71. KRENCHEL,HERBERT og J. BJØRNBAK-HANSEN: Undersøgelse af let konstruktionsbetons væsentligste materialeparametre. 1976.
R 72. BRÆSTRUP,M.W., M.P. NIELSEN, FINN BACH and B.CHR. JENSEN: Shear tests on reinforced concrete T-beams. Series T. 1976.
R 73. NIELSEN,M.P. and M.W. BRÆSTRUP: Plastic shear strength of reinforced concrete beams. 1976.
R 74. Resume-oversigt 1975. Summaries of papers 1975. 1976.
R 75. BRÆSTRUP,M.W., M.P. NIELSEN, B.CHR. JENSEN and FINN BACH: Axisymmetric punching of plain and reinforced concrete. 1976.
R 76. BRØNDUM-NIELSEN,TROELS: Partial prestressing. 1976.
R 77. LAURSEN, M.E.: EQSHELL. An equilibrium shell finite element for arbitrary shell geometry. 1977.
R 78. DYRBYE, CLAËS and FLEMMING BLIGAARD PEDERSEN: Rubber mountings and vibration isolation. 1977.
R 79. BRÆSTRUP, M.W., M.P. NIELSEN and FINN BACH: Rational analysis and design of stirrups in reinforced concrete beams. 1977.