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ON THE BEHAVIOUR OF CRACKED REINFORCED CONCRETE BEAMS IN THE ELASTIC RANGE

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ABSTRACT

The stiffness of reinforced concrete beams under combined shear and bending and under torsion is studied after fully development of cracking. In both cases the method involves minimizing of the complementary, potential energy for a class of the statically admissible stress-distributions. Comparison with tests from the litterature confirmes the applicability of the method. For beams subjected to combined shear and bending, the results are employed to calculate safe limits for the permissible strut inclination used in shear design based on the truss analogy with variable strut inclination. Here it is required that yielding of the stirrups do not occur under serviceload. For beams subjected to torsion it is demonstrated, how the method leads to a usefull lower bound estimation of the stiffness.

RESUMÉ.

Jernbetonbjælkers stivhed ved bøjning med forskydning og ved vridning studeres i det fuldt revnede studium. Den benyttede metode involverer i begge tilfælde minimering af den komplementære, potentielle energi for en delmængde af de statisk tilladelige spændingsfordelinger. Ved sammenligning med forsøg fra litteraturen bekræftes metodens anvendelighed. For bjælker udsat for bøjning med forskydning benyttes resultaterne til at sætte sikre grænser for, hvor små betontrykhældninger, der her tillades anvendt under dimensionering af forskydningsarmeringen efter gitteranalogien med variabel trykhældning, når det kræves, at der ikke opstår flydning i bøjlerne i brugsstadiet. For bjælker udsat for vridning vises, hvorledes energibetragtningerne leder frem til en udmærket nedreværdi-bestemmelse af stivheden.

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INTRODUCTION

It has always been a matter of great difficulty to describe the behaviour of reinforced concrete members in shear or torsion after the development of cracks. The purpose of the present report is to demonstrate, how valuable information about such members can be derived by means of simple energy considerations.

NOTATIONS

A_{al}	Cross sectional area of longitudinal reinforcement
A_{as}	Stirrup area crossing the concrete area $b\ c$
a	Length of shear span
b	Width of beam
C	Compressive stringer force
c	Stirrup spacing
E	Elastic energy
E_a	Elastic modulus of reinforcement
h	Depth of beam
M	Bending moment
P	Applied load
P_s	Service load
P_u	Ultimate load
T	Tensile stringer force
V	Torsional moment
x, y	Coordinates in a rectangular system
y_o	Depth of compression zone
α	Torsion per unit length of beams
θ	Inclination of diagonal concrete compression in the web
κ	$\cot\theta$
κ_{max}	Maximum permissible value of κ in design calculations
ρ_{as}	Geometrical degree of stirrup reinforcement
σ_{al}	Stress in longitudinal reinforcement
σ_{as}	Stress in stirrup reinforcement
σ_b	Compressive stress in concrete
σ_c	Compressive strength of concrete
σ_{fl}	Yield stress of longitudinal reinforcement
σ_{fs}	Yield stress of stirrup reinforcement
$\sigma_x, \sigma_y, \tau_{xy}$	Equivalent stirrup stresses
τ_b	Shear stress in concrete
φ	Inclination of compression concrete stress at torsion

1. BEAMS SUBJECTED TO SHEAR AND BENDING

When reinforced concrete beams are designed in the ultimate state, using the theory of plasticity, it is possible to achieve full utilization of both shear reinforcement and concrete. This design might, however, under certain circumstances, lead to so small an amount of shear reinforcement that yielding of this occurs long before the failure load is reached, and perhaps already under service load, which would cause unacceptable crack widths.

In this part of the report, beams subjected to shear and bending are considered in the elastic state but after cracking of the concrete by setting up a statically admissible stress field depending on one parameter and minimizing the complementary potential energy with respect to this parameter. Using this method it appears that the forces in the transverse reinforcement can be estimated, and it is demonstrated how these forces can be limited under service load by introducing limits of the permissible inclination of the concrete stresses in the web used in the plastic ultimate design. Such limits correspond to a required minimum of shear reinforcement. The necessary limits to avoid yielding of the stirrups under service loads are found for different types of transverse and longitudinal reinforcement.

For readers interested in further details, attention is drawn to the fact that this chapter is based on a master thesis by Verner Jensen, see [77.1].

1.1. Calculation of stirrup stresses.

Consider a horizontal, simply supported beam subjected to two concentrated loads, P, applied symmetrically at the distance a from the supports, see fig. 1.

For this case, Nielsen [67.1] has developed a statically admissible stress field, sketched in fig. 2, where a part of the shear zone is shown.

In the web the concrete stress state is a uniaxial compression σ_b inclined the angle θ to the x-axis. The horizontal compression

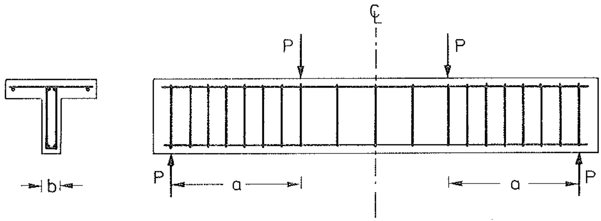


Fig. 1.

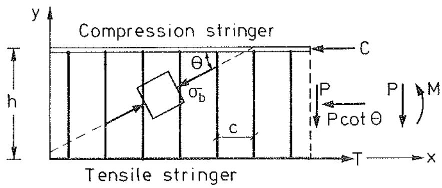


Fig. 2.

zone at the top of the beam is idealized as a stringer carrying a force C and the tensile reinforcement as a stringer carrying a force T. The stirrup forces are transformed to an equivalent stirrup stress, σ_y, distributed over the concrete area. If the stirrup stress is σ_{as}, the equivalent stirrup stress is

$$\sigma_y = \frac{A_{as}\sigma_{as}}{cb} = \rho_{as}\sigma_{as} \qquad (1)$$

where A_{as} is the stirrup area crossing the concrete area $c \cdot b$, and c is the longitudinal stirrup spacing. ρ_{as} is the geometrical degree of reinforcement. Since the stirrups are vertical the equivalent stirrup stresses σ_x and τ_{xy} are zero.

To be statically admissible, the stress field has to fullfil the boundary conditions and the equations of equilibrium. For detailed examination, the reader is referred to [67.1]. Here we only quote the resulting stresses and stringer forces:

$$\sigma_b = \frac{P}{bh}(\text{tg}\ \theta + \cot\ \theta) \tag{2}$$

$$\sigma_{as} = \frac{P}{\rho_{as} bh}\text{tg}\ \theta \tag{3}$$

$$C = \frac{M}{h} - \tfrac{1}{2}P \cot\ \theta \tag{4}$$

$$T = \frac{M}{h} + \tfrac{1}{2}P \cot\ \theta \tag{5}$$

Denoting the cross sectional area of the longitudinal reinforcement as A_{al} , the stresses in this reinforcement along the x-axis are

$$\sigma_{al} = \frac{T}{A_{al}} = \frac{1}{A_{al}}\left(\frac{P\ x}{h} + \tfrac{1}{2}P \cot\ \theta\right) \tag{6}$$

x being zero at the support.

If the load is acting at the top of the beam, the stress distribution above is rather improbable in the section under the load. Therefore, we introduce a modified stress distribution, see fig. 3. The shaded region on fig. 3 is in equilibrium under the applied loads, provided that it carries the vertical load P and the part $\tfrac{1}{2}P \cot\ \theta$ of the compressive stringer force, see also [67.1]. It should be noticed that the stress state in the region considered is not homogeneous.

With the modified stress distribution, the tensile stringer force does not exceed the value

$$T = P\frac{a}{h} \tag{7}$$

in any part of the beam.

Under service load we now estimate the inclination of the compressive

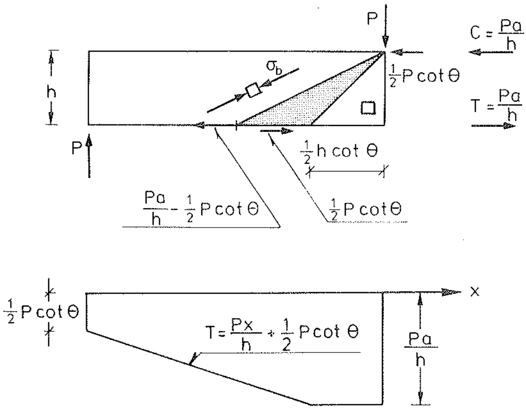


Fig. 3.

stress in the web by minimizing the complementary potential energy. Assuming linear elasticity and Poissons ratio being zero for both concrete and reinforcement, the complementary energy is equal to the elastic energy.

Using the modified stress distribution, it is only necessary to consider the elastic energy of the shear span and, furthermore, numerical calculations (see [77.1]) show that the contribution from the concrete can be disregarded. This means that in order to find $\operatorname{tg}\theta$ we only need to minimize the expression

$$E = \int_o^a \frac{1}{2E_a}[hb\rho_{as}\sigma_{as}^2 + A_{al}\sigma_{al}^2]dx \tag{8}$$

where E_a is the elastic modulus of the reinforcement. When A_{al} and ρ_{as} are constant over the whole shear span, the requirement

$$\frac{dE}{d\theta} = 0 \tag{9}$$

leads to the equation

$$tg^5\theta - \frac{a\ b\ \rho_{as}}{4\ A_{al}}\ tg^2\theta + \frac{h^2\ b\ \rho_{as}}{16a\ A_{al}} = 0 \tag{10}$$

which can be solved by iteration using the formula

$$tg\ \theta = \sqrt[5]{\frac{h^2\ b\ \rho_{as}}{16\ a\ A_{al}}(4\ \frac{a^2}{h^2}\ tg^2\theta - 1)} \tag{11}$$

If the shear span is not too short, for instance if $\frac{a}{h} > 3$, only neglectible errors will be introduced using the solution

$$tg\ \theta = \sqrt[3]{\frac{a\ b\ \rho_{as}}{4\ A_{al}}} \tag{12}$$

instead of the iteration formula (11).

After calculating $tg\ \theta$ from (11) or (12), the stirrup stresses are determined from (3) until

$$P = \frac{bh\ \rho_{as}\sigma_{fs}}{tg\ \theta} \tag{13}$$

where the stirrups start yielding. σ_{fs} is the yield stress of the stirrup reinforcement. If the load is increased further, the stirrup stress remains constant at

$$\sigma_{as} = \sigma_{fs} \tag{14}$$

until the failure load of the beam is reached.

In fig. 4-7, stirrup forces calculated from the above theory are compared with the actual stirrup forces measured by means of strain-gauges under shear tests with T-beams carried out at the Structural Research Laboratory at the Technical University of Denmark [76.1]. From the figures it is seen that the stirrups do not become active at lower loads due to the tensile strength of the concrete, which is not included in the theoretical solution. Apart from this, the theory agrees well with the tests.

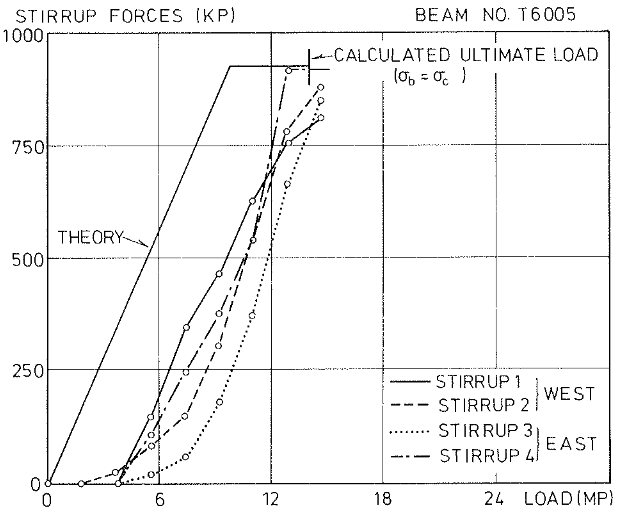

STIRRUP FORCES (KP)
BEAM NO. T6005
CALCULATED ULTIMATE LOAD
($\sigma_b = \sigma_c$)
THEORY
1000
750
500
250
0
0
6
12
18
24
LOAD (MP)
STIRRUP 1
STIRRUP 2
WEST
STIRRUP 3
STIRRUP 4
EAST

Fig. 4.

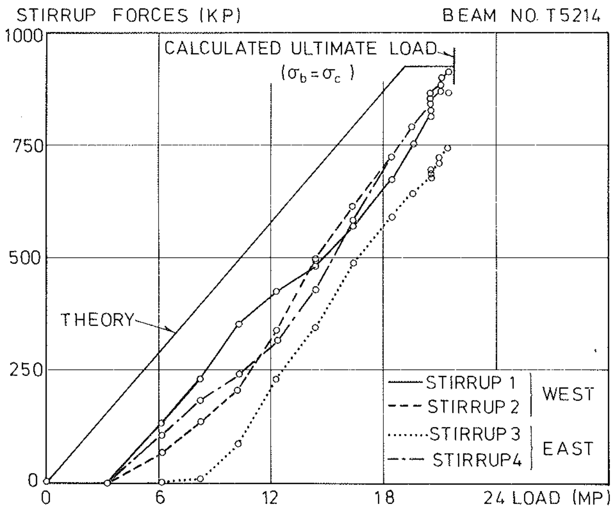
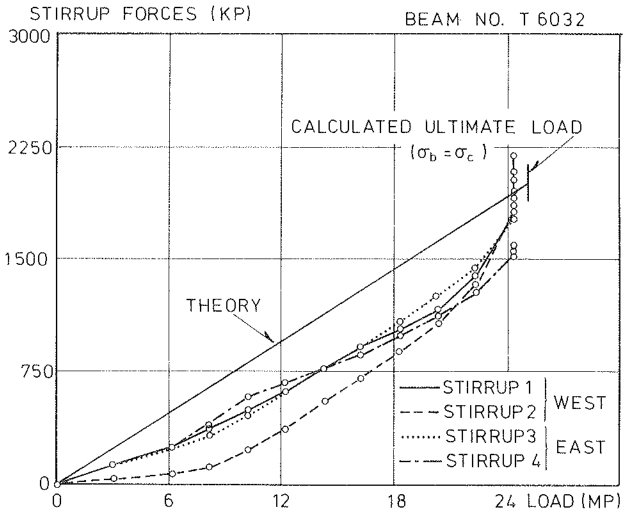
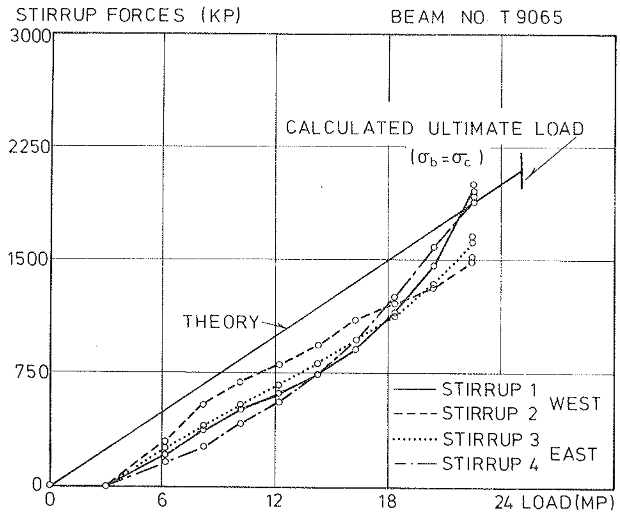

STIRRUP FORCES (KP)
BEAM NO. T5214
CALCULATED ULTIMATE LOAD
($\sigma_b = \sigma_c$)
THEORY
1000
750
500
250
0
0
6
12
18
24
LOAD (MP)
STIRRUP 1
STIRRUP 2
WEST
STIRRUP 3
STIRRUP 4
EAST

Fig. 5.



STIRRUP FORCES (KP)
BEAM NO. T 6032
3000
2250
1500
750
0
0
6
12
18
24 LOAD (MP)
CALCULATED ULTIMATE LOAD
($\sigma_b = \sigma_c$)
THEORY
STIRRUP 1
STIRRUP 2
WEST
STIRRUP3
STIRRUP 4
EAST

Fig. 6.



STIRRUP FORCES (KP)
BEAM NO T 9065
3000
2250
1500
750
0
0
6
12
18
24 LOAD(MP)
CALCULATED ULTIMATE LOAD
($\sigma_b = \sigma_c$)
THEORY
STIRRUP 1
STIRRUP 2
WEST
STIRRUP 3
STIRRUP 4
EAST

Fig. 7.

1.2. Design applications.

Future design of reinforced concrete beams subjected to shear can be expected to be based upon the theory of plasticity, see Nielsen et al [76.2]. If, for instance, we consider the beam type treated in section 2, coinciding upper and lower bound solutions are presented in [76.2], where the theory is compared with test results from about 200 shear tests, and excellent agreement is found. The lower bound solution mentioned can be derived using the stress distribution from section 1.1, and we find that if

$$\frac{P_u}{b\,h}\left(\kappa + \frac{1}{\kappa}\right) = \sigma_b \leq \sigma_c \tag{15}$$

then the shear capacity is determined by

$$P_u = \kappa \; b \; h \; \rho_{as} \sigma_{fs} \tag{16}$$

where $\kappa = \cot\theta$ indicates the inclination of the concrete stresses in the web at the ultimate load. From (15) and (16) it is seen that the best lower bound solution is obtained when $\sigma_b = \sigma_c$, which means that

$$\kappa = \sqrt{\frac{\sigma_c - \rho_{as}\sigma_{fs}}{\rho_{as}\sigma_{fs}}} \tag{17}$$

However, design based on (16) and (17) could result in an amount of stirrup reinforcement so small that yielding of the stirrups might occur under service load. To avoid this we introduce an upper limit for the permissible value of κ by means of the results from section 2. Here we found the stirrup stresses before yielding to be

$$\sigma_{as} = \frac{P}{\rho_{as}\,b\,h}\;\mathrm{tg}\;\theta = \frac{P}{\rho_{as}\,b\,h}\;\sqrt[3]{\frac{a\;b\;\rho_{as}}{4\;A_{al}}} \tag{18}$$

These stresses reach a maximum value if A_{al} is as small as possible, so the most severe case corresponds to designing the longitudinal reinforcement by means of the formula

$$A_{al} = \frac{P_u\;a}{\sigma_{fl}\;h} \tag{19}$$

where σ_{fl} is the yield stress of the longitudinal reinforcement. From (16), plastic design leads to a sufficient amount of stirrup reinforcement, namely

$$\rho_{as} = \frac{P_u}{b\,h\,\kappa\,\sigma_{fs}} \tag{20}$$

if (15) is fullfilled.

Inserting (19) and (20) into (18) the stirrup stresses under service load, P_s , can be written as

$$\sigma_{as} = \frac{P_s}{P_u}\,\kappa\,\sigma_{fs}\sqrt[3]{\frac{\sigma_{fl}}{4\kappa\,\sigma_{fs}}} \tag{21}$$

Under service load we require that

$$\sigma_{as} < \sigma_{fs} \tag{22}$$

which, together with (21), leads to the following upper limit of κ:

$$\kappa_{max} < \sqrt{\frac{4\sigma_{fs}}{(\frac{P_s}{P_u})^3\,\sigma_{fl}}} \tag{23}$$

On fig. 8a, the limits found from (23) are illustrated graphically.

Similar calculations can be carried out for other loading cases and for other types of reinforcement. Some results are shown on the figures 8b - 8f.

The requirements to κ_{max} stem from serviceability considerations. The ratio P_u/P_s depends on the load factors and the partial safety factors on the material strengths. In Denmark one may generally assume that

$$\frac{P_u}{P_s} \geq 1,6 \tag{24}$$

and that σ_{fs}/σ_{fl} is not essentially lower than ½.

From the figures 8a - 8f it then appears that suitable overall limits for κ_{max} can be chosen to be

$$\kappa_{max} \leq \begin{cases} 2{,}5 & \text{for beams with constant longitudinal reinforcement} \\ 2{,}0 & \text{for beams with cut off longitudinal reinforcement} \end{cases} \qquad (25)$$

In selecting those values, proper account has been taken to the fact that the calculation of the stirrup stresses carried out as described will be on the safe side, cf. fig. 4-7.

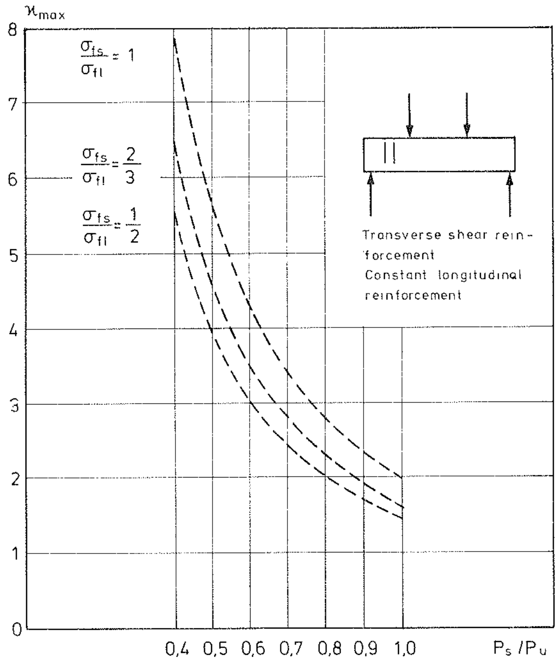


Fig. 8a.

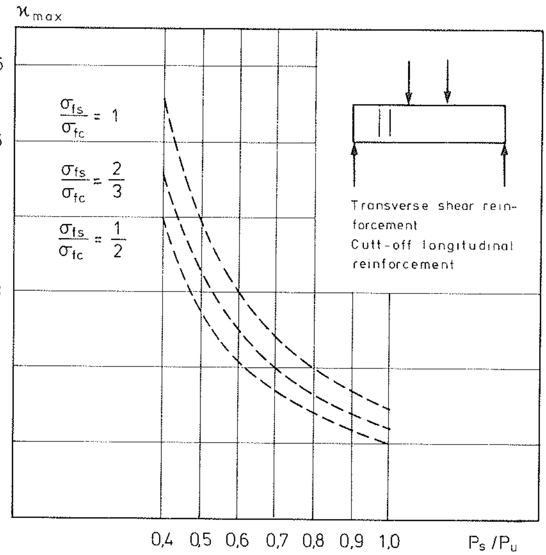

$\varkappa_{max}$
6
5
4
3
2
1
$\frac{\sigma_{fs}}{\sigma_{fc}} = 1$
$\frac{\sigma_{fs}}{\sigma_{fc}} = \frac{2}{3}$
$\frac{\sigma_{fs}}{\sigma_{fc}} = \frac{1}{2}$
Transverse shear rein-
forcement
Cutt-off longitudinal
reinforcement
0,4 0,5 0,6 0,7 0,8 0,9 1,0
P_s / P_u

Fig. 8b.

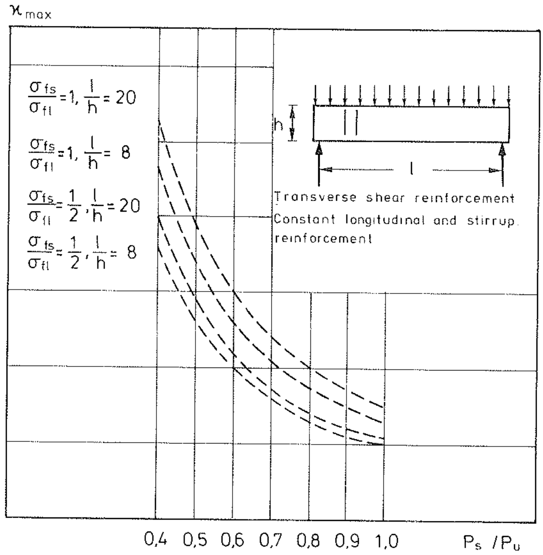

$\varkappa_{max}$
6
5
4
3
2
1
0
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \frac{l}{h} = 20$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \frac{l}{h} = 8$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \frac{l}{h} = 20$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \frac{l}{h} = 8$
h
l
Transverse shear reinforcement
Constant longitudinal and stirrup
reinforcement
0,4 0,5 0,6 0,7 0,8 0,9 1,0
P_s / P_u

Fig. 8c.

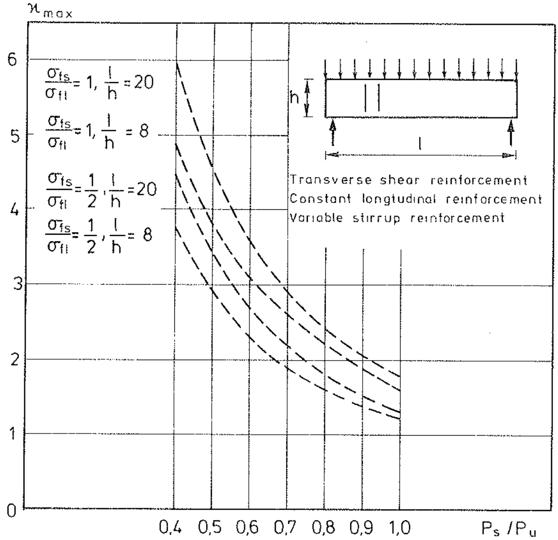

$\varkappa_{max}$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \frac{l}{h} = 20$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \frac{l}{h} = 8$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \frac{l}{h} = 20$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \frac{l}{h} = 8$
Transverse shear reinforcement
Constant longtudinal reinforcement
Variable stirrup reinforcement
h
l
0,4 0,5 0,6 0,7 0,8 0,9 1,0
P_s / P_u

Fig. 8d.

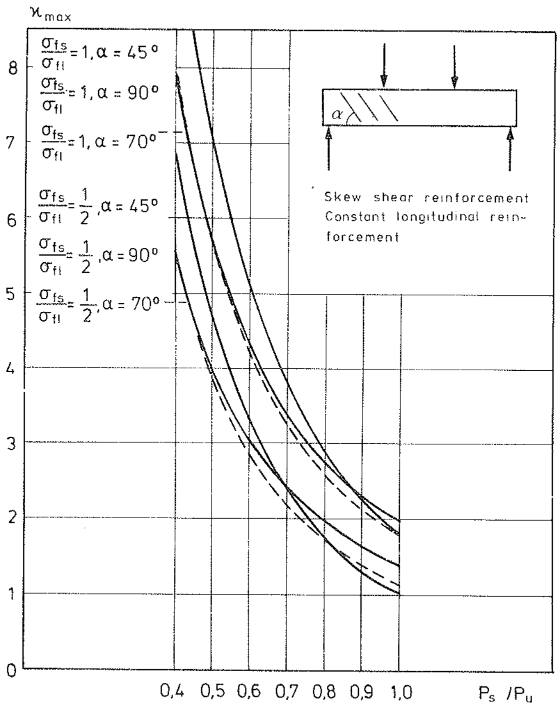
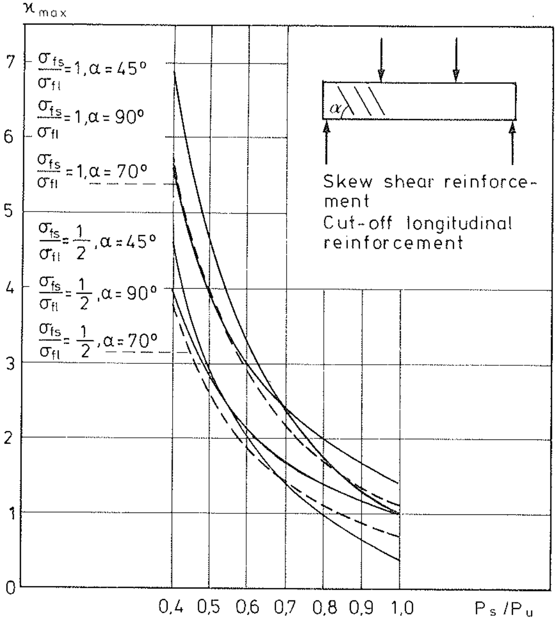

$\varkappa_{max}$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 45°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 90°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 70°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 45°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 90°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 70°$
α
Skew shear reinforcement
Constant longitudinal rein-
forcement
0,4 0,5 0,6 0,7 0,8 0,9 1,0
P_s / P_u

Fig. 8e.



$\varkappa_{max}$
7
6
5
4
3
2
1
0
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 45°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 90°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = 1, \alpha = 70°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 45°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 90°$
$\frac{\sigma_{fs}}{\sigma_{fl}} = \frac{1}{2}, \alpha = 70°$
α
Skew shear reinforcement
Cut-off longitudinal reinforcement
0,4 0,5 0,6 0,7 0,8 0,9 1,0
P_s/P_u

Fig. 8f.

2. BEAMS SUBJECTED TO TORSION

In the following we shall consider the behavior of cracked reinforced concrete beams in torsion. It will be demonstrated how a reasonable lower bound estimate of the stiffness can be derived by means of energy considerations. The method used is, as in chapter 1, based on the principle of minimum of the complementary potential energy.

The content of this chapter is based on parts of a master thesis made by H.H. Christensen [77.2].

2.1. Stiffness of the cracked section.

In this chapter we shall deal with reinforced concrete beams with rectangular cross-section and vertical stirrups, as shown on fig. 9.

As a statically admissible stress field the one illustrated in fig. 10 is chosen.

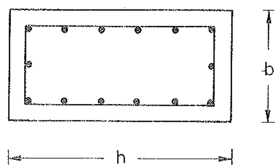
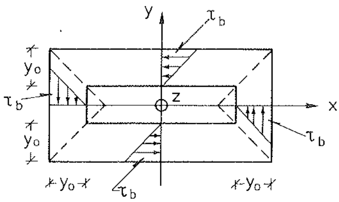
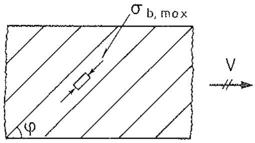


Fig. 9.





Shear stresses in a cross-section

Concrete stresses at the surface of the beam.

Fig. 10.

The stress field in the concrete is a uniaxial compression inclined at the angle φ to the center axis of the beam. The concrete stresses reach a maximum value, $\sigma_{b,max}$, at the surface of the beam and then decrease lineary towards zero along the distance y_o from the surface. The concrete is assumed to have no tensile strength, so that at larger distances than y_o from the surface, the concrete stresses are zero. As in chapter 1, the stirrup forces are transformed to an equivalent stirrup stress.

Denoting the torsional moment as V , the equilibrium conditions now lead to the following stresses in the concrete and the reinforcement:

$$\sigma_{b,max} = \frac{V(1 + tg^2\varphi)}{y_o(hb - \frac{2}{3}(h + b)y_o + \frac{2}{3} y_o^2)\, tg\, \varphi} \qquad (26)$$

$$\sigma_{al} = \frac{V((h + b) - \frac{4}{3} y_o)}{A_{al}(hb - \frac{2}{3}(h + b)y_o + \frac{2}{3} y_o^2)\, tg\, \varphi} \qquad (27)$$

$$\sigma_{as} = \frac{V \cdot c\ tg\ \varphi}{2A_{as}(hb - \frac{2}{3}(h + b)y_o + \frac{2}{3} y_o^2)\, tg\, \varphi} \qquad (28)$$

From these stresses the complementary potential energy under the same assumptions as in chapter 1 can be determined as the elastic energy. The elastic energy per unit length of the beam is found to be:

$$E = 2 \int_{\frac{b}{2} - y_o}^{\frac{b}{2}} \frac{\sigma^2_{b,max} (y - (\frac{b}{2} - y_o))^2}{2E_b \, y_o^2} (2y + (h - b + 2y)) dy$$

$$+ \frac{\sigma^2_{al} \, A_{al}}{2E_a} + \frac{\sigma^2_{as} \, A_{as} \, \frac{1}{2}s}{2E_a \, c} \qquad (29)$$

where s is the circumferencial length of a stirrup.

The above expression for the elastic energy now has to be minimized with respect to y_o and $tg\,\varphi$. Using partial differentiation, we are lead to two coupled equations of fifth order, which have to be solved numerically. These calculations are not included here. Having determined y_o and $tg\,\varphi$, these values are inserted in (29), and the torsion of the beam can then be found using the Clapeyron principle, which in this case yields

$$\frac{1}{2} V \alpha = E$$

where α is the torsion per unit length of the beam.

On figure 12 the theory above is compared with results from tests made by P. Lampert and B. Thürlimann, [68.1]. The cross-section of the beams tested are shown on fig. 11.

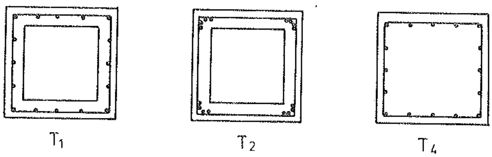


Fig. 11.

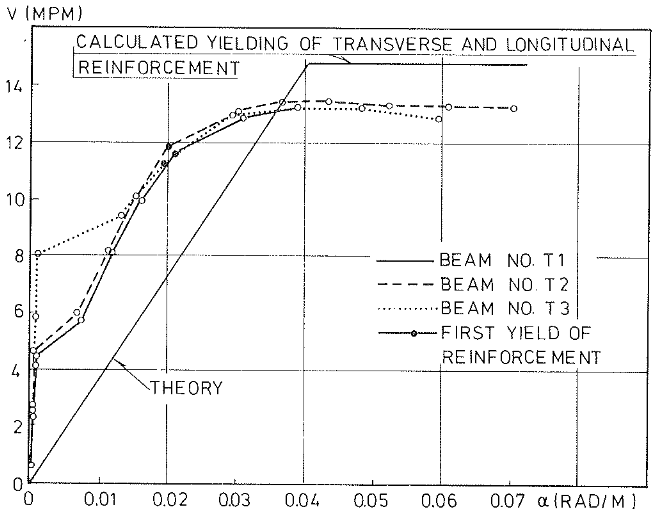


Fig. 12.

All the beams had the same amount of reinforcement, but the beam T2 had a different arrangement of the longitudinal reinforcement, a difference which does not seem to affect the behavior of the beam in comparison with No. T 1.

The cross-sections of beams Nos. T 1 and T 2 were hollow, while T 4 had a massive cross section. We see that this circumstance did not make any significant difference to the stiffness after the concrete was cracked, as expected since the depth of the compression zone, y_o , according to the theory is calculated to be smaller than the wall thickness of the hollow cross-section.

From fig. 12 it is seen that the theory gives a usefull lower bound estimate of the stiffness of the beams.

CONCLUSIONS.

By comparing the theoretical results with tests, it appears that the method used leads to reasonable, safe calculations of stresses or deformations for the types of cracked reinforced concrete members considered.

For beams under combined shear and bending it is demonstrated that when the ultimate sheer design is based on the truss analogy with variable strut inclination, then proper limitation of the deformations of the stirrups under the service load can be ensured by requirering

$$\kappa_{max} = \cot\Theta \leq \begin{cases} 2{,}5 & \text{for constant longitudinal reinforcement} \\ 2{,}0 & \text{for cut off longitudinal reinforcement} \end{cases}$$

Here Θ denotes the inclination of the concrete struts used under the ultimate design mentioned.

In the case of reinforced concrete beams undergoing torsion, safe determination of the stiffness after cracking of the concrete can be derived using the expounded method, but for the use in practice, simplifications of the calculations would be desirable.

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