AFDELINGEN FOR

BÆRENDE KONSTRUKTIONER

DANMARKS TEKNISKE HØJSKOLE

STRUCTURAL RESEARCH LABORATORY

TECHNICAL UNIVERSITY OF DENMARK

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STRESS ANALYSIS OF CONCRETE SECTIONS UNDER SERVICE LOAD

RAPPORT NR. R 109 1979

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Reprint of paper published in the
Journal of the American Concrete
Institute, Proc. Vol. 76, No. 2,
February, 1979.

ACI JOURNAL TECHNICAL PAPER

Title no. 76-10

Stress Analysis of Concrete Sections Under Service Load

by Troels Brøndum-Nielsen

The stress analysis of concrete sections under service load is complicated by the interaction of shrinkage, creep, elastic strains of concrete and steel, and steel relaxation. Further complications arise in the case of partially prestressed concrete structures, with cracked cross sections under service load and with mixed reinforcement consisting of a combination of prestressed and nonprestressed reinforcement with individual mechanical properties and different effective depths. Combinations of sustained and instantaneous loads also complicate the analysis, whereas compound bending is easily taken into account. An analysis is suggested covering these problems in cases of symmetrical bending.

Keywords: bending; **cracking (fracturing)**; **creep properties**; fatigue (materials); loads (forces); partial prestressing; prestressed concrete; prestressing steels; reinforced concrete; **shrinkage**; strains; **stress analysis**; structural analysis.

Received Apr. 11, 1978, and reviewed under Institute publication policies. Discussion closes May 1, 1979.

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EQUILIBRIUM CONDITIONS

The concrete tensile strength is neglected. Under service load plane cross-sections are assumed to remain plane and concrete stresses are assumed to be proportional to the corresponding concrete strains exclusive of shrinkage strain.

For the concrete, compressive stresses, σ_c, are taken as positive and the shrinkage strain, ε_{cs}, corresponding to reduction in length is taken as positive. For the reinforcement, tensile stresses and strains corresponding to elongation are taken as positive.

For the case of an arbitrary cross-section, sustained pure uniaxial bending and same effective depths, d, of both types of reinforcement (Fig. 1), moment equilibrium with respect to the reinforcement requires:

$$M = \frac{\sigma_c}{\beta d} \int_0^{\beta d} y\, b(y) \left[(1-\beta)d + y\right] dy = \frac{\sigma_c}{\beta d} \left[S(1-\beta)d + I\right] \qquad (1)$$

with the notation:

- y: distance above neutral axis
- $b(y)$: width of cross-section at distance y above neutral axis
- M: sustained bending moment. Positive sign corresponds to compression in top fibres
- S: first moment of area of compressed concrete section with respect to the neutral axis
- σ_c: maximum concrete compressive stress
- d: effective depth
- βd: depth of neutral axis
- I: second moment of area of compressed concrete section with respect to the neutral axis

The neutral axis is the line at which the concrete stresses are equal to zero, but where, due to shrinkage, the concrete strains are not necessarily equal to zero.

Equilibrium in the axial direction requires:

$$A_s \varepsilon_s E_s + A_p \sigma_p - \frac{\sigma_c}{\beta d} \int_0^{\beta d} y\, b(y) dy = A_s \varepsilon_s E_s + A_p \sigma_p - \sigma_c \frac{S}{\beta d} = 0 \quad (2)$$

with the notation:

A_s : cross-sectional area of non-prestressed steel

ε_s : strain in non-prestressed steel

E_s : strain modulus of non-prestressed steel

A_p : cross-sectional area of prestressing steel

σ_p : tensile stress in prestressing steel

RELAXATION

The relaxation of the prestressing steel is taken into account by assuming a stress-strain relationship for constant, sustained load giving σ_p as a function of the corresponding strain ε_p at a given time:

$$\sigma_p = f(\varepsilon_p) \quad (3)$$

CREEP

For the concrete, the following assumptions are made both for the initial strain and for the creep strains created by sustained stresses (shrinkage strain not included):

The strain is assumed to be proportional to the corresponding concrete stress.

Strain contributions due to stress contributions applied at different times are assumed to be additive (law of superposition).

These assumptions are illustrated in Fig. 2, where the following notation is used:

ε_{ci} : instantaneous concrete strain

ε_{cc} : concrete strain due to creep

E_{cq} : strain modulus of concrete for instantaneous load

E_{cg} : effective strain modulus of concrete for sustained load

COMPATIBILITY

Compatibility requires (Fig. 1):

$$\Delta \varepsilon_p + \varepsilon_{cs} = \varepsilon_s + \varepsilon_{cs} = \frac{\sigma_c}{E_{cg}} \frac{1-\beta}{\beta} \quad (4)$$

with the notation:

$\Delta\varepsilon_p$: strain increment in prestressing steel over pretensioning strain, ε_{po}

ε_{po} : strain in prestressing steel when the total strain (including shrinkage strain) in the concrete and in the non-prestressed reinforcement is equal to zero.

ε_{cs} : the shrinkage strain that would occur in the concrete if its stresses remained equal to zero and its temperature remained unchanged

The total strain in the prestressing steel is:

$$\varepsilon_p = \varepsilon_{po} + \Delta\varepsilon_p \tag{5}$$

ANALYSIS PROCEDURE

The results of the above derivation may conveniently be presented in the following form and sequence:

Eq. (1):

$$\sigma_c = \frac{M\beta d}{I + S(1-\beta)d} \tag{6}$$

Eq. (4):

$$\varepsilon_s = \frac{\sigma_c}{E_{cg}} \frac{1-\beta}{\beta} - \varepsilon_{cs} \tag{7}$$

Eqs. (5) and (4):

$$\varepsilon_p = \varepsilon_{po} + \varepsilon_s \tag{8}$$

Eq. (3):

$$\sigma_p = f(\varepsilon_p) \tag{9}$$

Introducing:

$$g(\beta) = A_s E_s \varepsilon_s + A_p \sigma_p - \sigma_c \frac{S}{\beta d} \tag{10}$$

Eqs. (2) and (10) yield:

$$g(\beta) = 0 \tag{11}$$

For an estimated value of β, the corresponding value of $g(\beta)$ may be calculated by successive substitution in Eqs. (6) to (10), and the β value can then be adjusted so as to satisfy Eq. (11) with sufficient accuracy.

If two values g_1 and g_2 of $g(\beta)$ have been calculated corresponding to the β values β_1 and β_2, linear interpolation or extrapolation leads to the following improved β value:

$$\beta_i = \frac{g_1 \beta_2 - g_2 \beta_1}{g_1 - g_2} \tag{12}$$

The method is suitable for programmable pocket calculators.

EFFECTIVE DEPTH

If the effective depth d_p of the prestressed reinforcement deviates from that of the non-prestressed reinforcement, d_s, an equivalent value

$$d = \frac{A_p \sigma_p d_p + A_s \sigma_s d_s}{A_p \sigma_p + A_s \sigma_s} \tag{13}$$

may be substituted for d with estimated values of σ_p and σ_s, which can be corrected at each step of the iteration.

In this case Eqs. (4), (7) and (8) should be modified as follows:

Eq. (4):

$$\varepsilon_s + \varepsilon_{cs} = \frac{\sigma_c}{E_{cg}} \left(\frac{d_s}{\beta d} - 1 \right) \tag{4a}$$

$$\Delta \varepsilon_p + \varepsilon_{cs} = \frac{\sigma_c}{E_{cg}} \left(\frac{d_p}{\beta d} - 1 \right) \tag{4b}$$

Eq. (7):

$$\varepsilon_s = \frac{\sigma_c}{E_{cg}} \left(\frac{d_s}{\beta d} - 1 \right) - \varepsilon_{cs} \tag{7a}$$

$$\Delta \varepsilon_p = \frac{\sigma_c}{E_{cg}} \left(\frac{d_p}{\beta d} - 1 \right) - \varepsilon_{cs} \tag{7b}$$

Eq. (8):

$$\varepsilon_p = \varepsilon_{po} + \Delta \varepsilon_p \tag{8a}$$

As the iteration is extended to cover variations of both β and d, the interpolation according to Eq. (12) may be suitably modified so as to apply to the product βd of β and d, i.e., if two values g_1 and g_2 of $g(\beta)$ have been calculated corresponding to the βd values $(\beta d)_1$ and $(\beta d)_2$, linear interpolation or extrapolation leads to the following improved βd value:

$$(\beta d)_i = \frac{g_1 (\beta d)_2 - g_2 (\beta d)_1}{g_1 - g_2} \tag{12a}$$

COMPOUND BENDING

In the case of compound bending corresponding to a normal force N, which is taken as positive when compressive, and which acts in the line of symmetry with the eccentricity e_p with respect to the reinforcement, this is equivalent to a pure bending moment

$$M = N e_p \tag{14}$$

plus an increase in the prestressing force from $A_p \sigma_p$ to $A_p \sigma_p + N$. e_p is taken as positive when N is located on the same side of the reinforcement as the compression zone.

If d_p deviates from d_s, e_p is taken as the distance from N to the resultant of the stresses in both types of reinforcement, i.e. to the point located at a distance d from the most compressed fibre. The distance d is given by Eq. (13).

The method thus also covers symmetric compound bending.

T-SECTIONS

For T-sections with a form as shown in Fig. 3 and for $\beta d > h$:

$$S = \frac{1}{2} b(\beta d)^2 - \frac{1}{2}(b-b_o)(\beta d - h)^2 \tag{15}$$

$$I = \frac{1}{3} b(\beta d)^3 - \frac{1}{3}(b-b_o)(\beta d - h)^3 \tag{16}$$

INSTANTANEOUS LOADS

The above stress analysis also covers the case of an instantaneous load, for instance, the situation immediately after tensioning of the tendons. In such cases, relaxation and creep do not occur. The following two modifications will consequently have to be introduced:

Eqs. (3) and (9) should read:

$$\sigma_p = E_p \varepsilon_p \tag{17}$$

where E_p denotes the strain modulus of the prestressing steel.

The strain modulus E_{cg} in Eqs. (4) and (7) should be substituted by the strain modulus, E_{cq}, corresponding to instantaneous stress variations.

COMBINATIONS OF SUSTAINED AND INSTANTANEOUS LOADS

The prestress and the dead load represent sustained load contributions. Consequently, instantaneous load contributions usually occur as an addition to a sustained load.

The stress analysis of such load combinations can be performed by a fictitious, instantaneous elimination of all concrete stresses from the sustained load (fictitious, instantaneous neutralization).

The sustained load corresponds to a normal force N_g with the eccentricity e_{pg} with respect to the reinforcement. This includes the special - but common - case of pure bending ($N_g = 0$, $e_{pg} = \infty$). The instantaneous load contribution changes the normal force N_g with eccentricity e_{pg} to N_q with eccentricity e_{pq} with respect to the reinforcement.

The corresponding stresses are found by an analysis analogous to that indicated above, (Eqs. (6) to (16)), with the following modifications:

To the stress contributions from the load N_q should be added the neutralized stresses in the non-prestressed and prestressed steel:

$$\sigma_{sn} = E_s\left[\varepsilon_s(1-\varkappa) - \varkappa\varepsilon_{cs}\right] \tag{18}$$

and

$$\sigma_{pn} = \sigma_p - \varkappa E_p(\varepsilon_s + \varepsilon_{cs}) \tag{19}$$

with the notation:

$$\varkappa = \frac{E_{cg}}{E_{cq}} \tag{20}$$

If d_p deviates from d_s, Eq. (19) should be modified as follows:

$$\sigma_{pn} = \sigma_p - \varkappa E_p(\Delta\varepsilon_p + \varepsilon_{cs}) \tag{19a}$$

FATIGUE

The suggested stress analysis is applicable for investigation of safety against fatigue. In this case, the average normal force over an extended period should be substituted for N_g.

For N_q, two values should be considered, corresponding to the upper and lower load extremes, which occur a given number of times.

The resulting maximum and minimum stresses in the concrete, in the prestressing steel and in the non-prestressed steel can then be compared with the corresponding results of fatigue tests for these materials so as to check whether there is sufficient safety against fatigue.

CEB-FIP RELAXATION FORMULA

The 1970 edetion of the CEB-FIP Recommendations* suggested a relaxation formula which can be expressed as follows:

For $\sigma_{po} \leqq \sigma_{p1}$ (σ_{po} denoting the initial value of σ_p): No relaxation:

$$\sigma_p = \sigma_{po} = E_p \varepsilon_p \quad \text{for} \quad \varepsilon_p \leqq \frac{\sigma_{p1}}{E_p} = \varepsilon_1 \qquad (21)$$

For $\sigma_{p1} \leqq \sigma_p \leqq \sigma_{p2}$:

Relaxation:

$$\Delta\sigma_p = E_r(\varepsilon_p - \varepsilon_1)^2 \qquad (22)$$

where E_r is a constant. The stress remaining after relaxation is

$$\sigma_p = E_p \varepsilon_p - \Delta\sigma_p = E_p \varepsilon_p - E_r(\varepsilon_p - \varepsilon_1)^2 \qquad (23)$$

Eqs. (21) and (22):

$$E_r = \frac{\Delta\sigma_p E_p^2}{(\sigma_{po} - \sigma_{p1})^2} \qquad (23a)$$

If $\Delta\sigma_p$ is known for a given value of σ_{po}, E_r can be calculated by Eq. (23a):

NUMERICAL EXAMPLE NO. 1

T-section (Fig. 3).

$b = 1.2$ m

$b_o = 0.2$ m

$h = 0.1$ m

$d = 0.5$ m

$A_s = 10^{-3}$ m^2

$A_p = 10^{-3}$ m^2

$E_{cg} = 10^4$ MPa

$E_s = 2.1 \cdot 10^5$ MPa

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* Comité Européen du Béton - Fédération Internationale de la Précontrainte: International recommendations for the design and construction of concrete structures. June 1970: FIP Sixth Congress, Prague, English edition, p. 25.

$$\varepsilon_{po} = 0.004$$

$$\varepsilon_{cs} = 0.0002$$

For instantaneous strain variations the strain modulus of the prestressing steel is

$$E_p = 2.1 \cdot 10^5 \text{ MPa}$$

The characteristic strength of the prestressing steel is

$$f_{ptk} = 1700 \text{ MPa}$$

$$\sigma_{p1} = 0.4\ f_{ptk}{}^{*}$$

$$\varepsilon_1 = \frac{\sigma_{p1}}{E_p} = \frac{0.4 \cdot 1700}{2.1 \cdot 10^5} = 0.00324$$

$$\sigma_{p2} = 0.75\ f_{ptk}$$

$$\varepsilon_2 = \frac{\sigma_{p2}}{E_p} = \frac{0.75 \cdot 1700}{2.1 \cdot 10^5} = 0.00607$$

For $\sigma_{po} = \sigma_{p2}$: $\Delta\sigma_p = 0.15\ \sigma_{p2}$

Eq. (23a):

$$E_r = \frac{0.15 \cdot 0.75 \cdot 1700 (2.1 \cdot 10^5)^2}{(0.75 - 0.4)^2 \cdot 1700^2} = 23.82 \cdot 10^6 \text{ MPa}$$

Eqs. (21) - (23):

For $\varepsilon_p \leqq 0.00324$:

$$\sigma_p = 2.1 \cdot 10^5\ \varepsilon_p$$

For $0.00324 \leqq \varepsilon_p \leqq 0.00607$:

$$\sigma_p = 2.1 \cdot 10^5\ \varepsilon_p - 23.8 \cdot 10^6 (\varepsilon_p - 0.00324)^2$$

or

$$\sigma_p = 364000\ \varepsilon_p - 23.8 \cdot 10^6\ \varepsilon_p^2 - 250 \qquad (9a)$$

* Some of the numerical values deviate from those suggested in the CEB-FIP Recommendations.

where the unit for σ_p is MPa. The corresponding stress-strain relationship is illustrated in Fig. 4.

The stress analysis will be carried out for the case of pure bending corresponding to a sustained bending moment M = 0.5 MNm.

Eqs. (15) and (16):

$$S = 0.15\beta^2 - \frac{1}{8}(\beta - 0.2)^2 \qquad (15a)$$

$$I = 0.05\beta^3 - \frac{1}{24}(\beta - 0.2)^3 \qquad (16a)$$

Eq. (10):

$$g(\beta) = 210\varepsilon_s + 10^{-3}\sigma_p - \sigma_c\frac{2S}{\beta} \qquad (10a)$$

The calculation is illustrated in Table 1.

If the same bending moment were applied at the time of tensioning the tendons, then the initial stress in the prestressing steel would be 972 MPa. This is found by an analogous analysis in which shrinkage, creep and relaxation are neglected. The loss of prestress is thus 20 MPa.

If only the creep is taken into account, but not shrinkage and relaxation, an analogous analysis leads to a stress of 988 MPa in the prestressing steel. The corresponding strain is 0.00471. If interaction were neglected, the relaxation would reduce the stress to 936 MPa according to Eq. (9a), and the shrinkage strain would reduce this stress to 894 MPa, corresponding to a total loss of prestress of 78 MPa instead of 20 MPa. This illustrates the considerable errrors that are introduced by neglecting the interaction of shrinkage, creep and relaxation.

NUMERICAL EXAMPLE NO. 2

For the cross-section treated in Numerical Example No. 1, the sustained bending moment M = 0.5 MNm is assumed to increase suddenly to 0.6 Mnm.

The value of $\varkappa$ in Eqs. (18)-(20) is taken as:

$$\varkappa = 0.3$$

Eq. (18):

$$\sigma_{sn} = 2.1 \cdot 10^5 [0.000820(1-0.3)-0.3 \cdot 0.0002]$$

$$\sigma_{sn} = 107.9 \text{ MPa}$$

Eq. (19)

$$\sigma_{pn} = 952 - 0.3 \cdot 2.1 \cdot 10^5 (0.000820 + 0.0002)$$

$$\sigma_{pn} = 888 \text{ MPa}$$

For both types of reinforcement the instantaneous strain and stress increments to be added to the neutralized values are:

$$\Delta\varepsilon_s = \Delta\varepsilon_p = \frac{\sigma_c}{E_{cq}} \frac{1-\beta}{\beta} = \frac{0.3\,\sigma_c}{10^4} \frac{1-\beta}{\beta} \qquad (7c)$$

and

$$\Delta\sigma_s = \Delta\sigma_p = E_s \Delta\varepsilon_s = E_p \Delta\varepsilon_p = 2.1 \cdot 10^5 \Delta\varepsilon_s \qquad (9b)$$

The total stresses in the two types of reinforcement are:

$$\sigma_s = \sigma_{sn} + \Delta\sigma_s = 107.9 + \Delta\sigma_s \qquad (24)$$

and

$$\sigma_p = \sigma_{pn} + \Delta\sigma_s = 888 + \Delta\sigma_s \qquad (25)$$

The equilibrium condition according to Eqs. (10) and (11) requires:

$$g(\beta) = A_s\sigma_s + A_p\sigma_p - \sigma_c \frac{S}{\beta d} = 10^{-3}(\sigma_s + \sigma_p) - \sigma_c \frac{S}{0.5\beta} \doteq 0 \qquad (26)$$

The calculation is illustrated in Table 2. In this case the interpolation value $\beta_i = 0.369$ is only used as a guide. A comparison with the values in Table 1 suggests that a slightly lower value should be estimated.

If the increase in the bending moment had had the character of a sustained load instead, then the stresses would have been: $\sigma_c = 13.41$ MPa, $\sigma_s = 309$ MPa and $\sigma_p = 1029$ MPa. The differences are most important in connexion with calculations of deformations or safety against fatigue.

NUMERICAL EXAMPLE NO. 3

This example illustrates the analysis for a case where the effective depth d_p of the prestressed reinforcement deviates from that of the non-prestressed reinforcement, d_s. The cross-section and all parameters are the same as in the numerical examples Nos. 1 and 2 with the sole difference that d_s is taken as equal to 0.6 m.

Eqs. (15) and (16):

$$S = 0.6(\beta d)^2 - 0.5(\beta d - 0.1)^2 \qquad (15b)$$

$$I = 0.4(\beta d)^3 - \frac{1}{3}(\beta d - 0.1)^3 \qquad (16b)$$

Eq. (7a):

$$\varepsilon_s = \frac{\sigma_c}{E_{cg}} \left(\frac{d_s}{\beta d} - 1 \right) - \varepsilon_{cs}$$

$$\sigma_s = 2.1 \cdot 10^5 \left[\frac{\sigma_c}{10^4} \left(\frac{0.6}{\beta d} - 1 \right) - 0.0002 \right]$$

$$\sigma_s = 21 \left[\sigma_c \left(\frac{0.6}{\beta d} - 1 \right) - 2 \right] \tag{27}$$

Eq. (7b):

$$\Delta \varepsilon_p = \frac{\sigma_c}{10^4} \left(\frac{0.5}{\beta d} - 1 \right) - 0.0002 \tag{28}$$

Eqs. (10) and (2):

$$g(\beta) = 10^{-3}(\sigma_s + \sigma_p) - \sigma_c \frac{S}{\beta d} = 0 \tag{26a}$$

The calculation is illustrated in Table 3.

The following values corresponding to $g(\beta) = 0$ are found by interpolating between the values of the last two columns:

σ_c = $\underline{9.82}$ MPa

σ_s = $\underline{182}$ MPa

σ_p = $\underline{910}$ MPa

$\Delta \varepsilon_p$ = 0.000527

Eq. (18):

$$\sigma_{sn} = 182(1 - 0.3) - 2.1 \cdot 10^5 \cdot 0.3 \cdot 0.0002 = 115 \text{ MPa}$$

Eq. (19a):

$$\sigma_{pn} = 910 - 0.3 \cdot 2.1 \cdot 10^5 (0.000527 + 0.0002) = 864 \text{ MPa}$$

Eqs. (24), (9b), (7a) and (7c) for $\varepsilon_{cs} = 0$

$$\sigma_s = \sigma_{sn} + E_s \frac{\sigma_c}{E_{cq}} \left(\frac{d_s}{\beta d} - 1 \right) = 115 + 2.1 \cdot 10^5 \frac{0.3\sigma_c}{10^4} \left(\frac{0.6}{\beta d} - 1 \right)$$

$$= 115 + 6.3\sigma_c \left(\frac{0.6}{\beta d} - 1 \right) \tag{29}$$

$$\sigma_p = 864 + 6.3\sigma_c \left(\frac{0.5}{\beta d} - 1 \right) \tag{30}$$

The calculation is illustrated in Table 4.

The following values corresponding to g(β) = 0 are found by interpolating between the values of the last two columns:

σ_c = $\underline{13.33}$ MPa

σ_s = $\underline{281}$ MPa

σ_p = $\underline{989}$ MPa

CONCLUSION

The suggested stress analysis of concrete sections under service load takes into account the interaction of the relevant physical properties such as creep, shrinkage, relaxation, etc.. It is deemed useful for calculating deformations, crack widths, safety against fatigue, etc.. As the analysis covers the general case of partially prestressed cross-sections cracked under service load, it includes the special cases of fully prestressed sections and reinforced concrete sections without prestress.

RECOMMENDATIONS

In reinforced and prestressed concrete cross-sections, variations in stress due to creep, shrinkage, relaxation, etc. are usually found simply by adding the individual contributions, which means that the interaction between them is neglected. The suggested analysis takes this interaction into account and thus avoids an inaccuracy that can lead to considerable errors.

TABLE 1 - Stress Analysis for Sustained Load

β			1.0	0.5	0.518	0.510
	Unit	Eq.				
S	m^3	15a	0.070	0.02625	0.02761	0.02700
σ_c	MPa	16a and 6	8.72	10.70	10.56	10.62
ε_s		7	-0.0002	0.000870	0.000783	0.000820
ε_p		8	0.0038	0.00487	0.00478	0.00482
σ_p	MPa	9a	790	958	946	952
$g(\beta)$	MN	10a	-0.4700	0.01720	-0.01529	-0.00027
β_i		12		0.518	0.510	0.510

TABLE 2 - Stress Analysis for Combination of Sustained and Instantaneous Loads

β			0.4	0.3	0.360
	Unit	Eq.			
S	m^3	15a	0.01900	0.01225	0.01624
σ_c	MPa	16a and 6	14.00	16.08	14.68
$\Delta\varepsilon_s$		7c	0.000630	0.001126	0.000783
$\Delta\sigma_s$	MPa	9b	132.3	236.5	164.4
σ_s	MPa	24	240.2	344.4	272.3
σ_p	MPa	25	1020	1124	1052
$g(\beta)$	MN	26	-0.0698	+0.1552	-0.000162
β_i		12		0.369	0.360

TABLE 3 - Stress Analysis for Sustained Load

β			0.55	0.556	0.557
	Unit	Eq.			
d	m		0.52	0.517	0.517
βd	m		0.286	0.287	0.288
S	m^3	15b	0.0318	0.0319	0.0321
σ_c	MPa	16b & 6	9.76	9.83	9.80
σ_s	MPa	27	183	183	181
$\Delta\varepsilon_p$		28	0.000530	0.000530	0.000521
ε_p		8a	0.00453	0.00453	0.00452
σ_p	MPa	9a	911	911	909
$g(\beta)$	MN	26a	0.0088	0.0014	-0.0023
d	m	13	0.517	0.517	0.517
$(\beta d)_i$	m	12a		0.287	

TABLE 4 - Stress Analysis for Combination of Sustained and Instantaneous Loads

β			0.39	0.387	0.386
	Unit	Eq.			
d	m		0.52	0.522	0.522
βd	m		0.203	0.202	0.201
S	m^3	15b	0.01942	0.01928	0.01914
σ_c	MPa	16b & 6	13.32	13.30	13.34
σ_s	MPa	29	279	280	282
σ_p	MPa	30	987	988	989
$g(\beta)$	MN	26a	-0.0083	-0.0014	+0.0007
d	m	13	0.522	0.522	0.522
$(\beta d)_i$	m	12a		0.202	

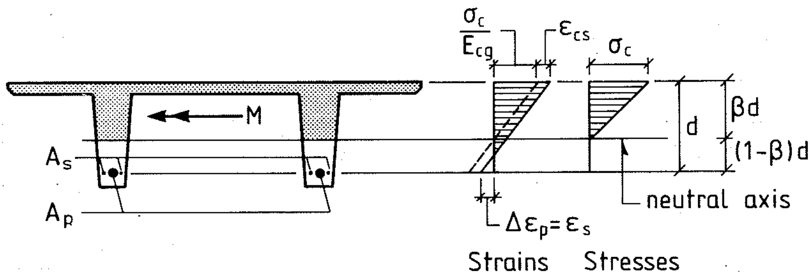

FIG. 1. Cracked Concrete Cross-Section

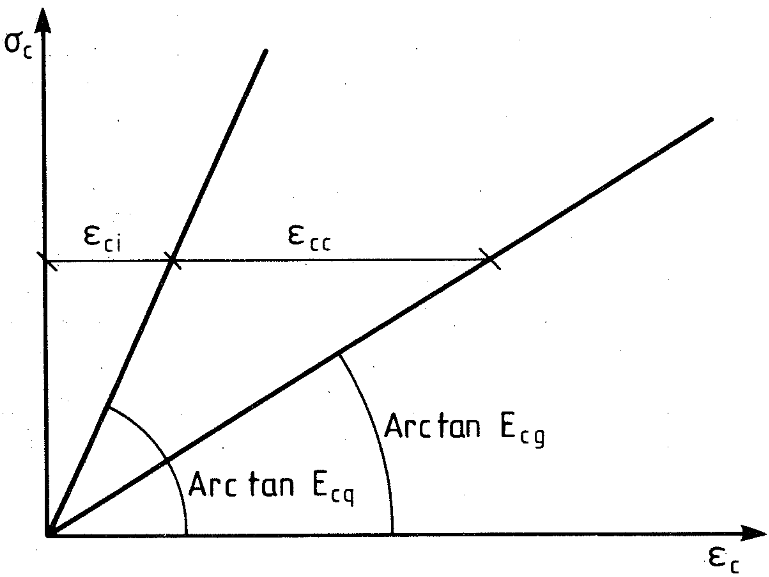

FIG. 2. Stress-Strain Curves for Concrete

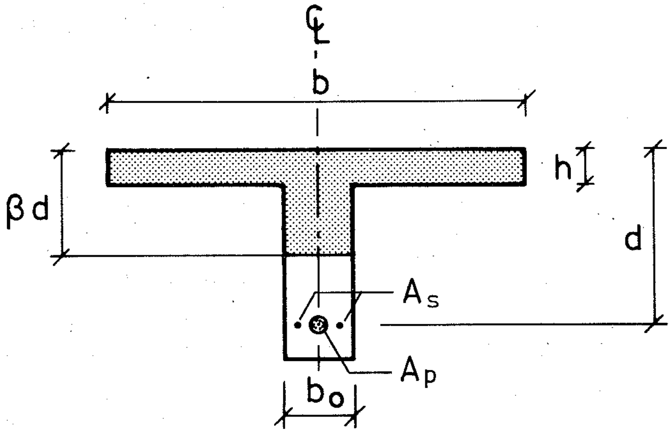

FIG. 3. T - Section

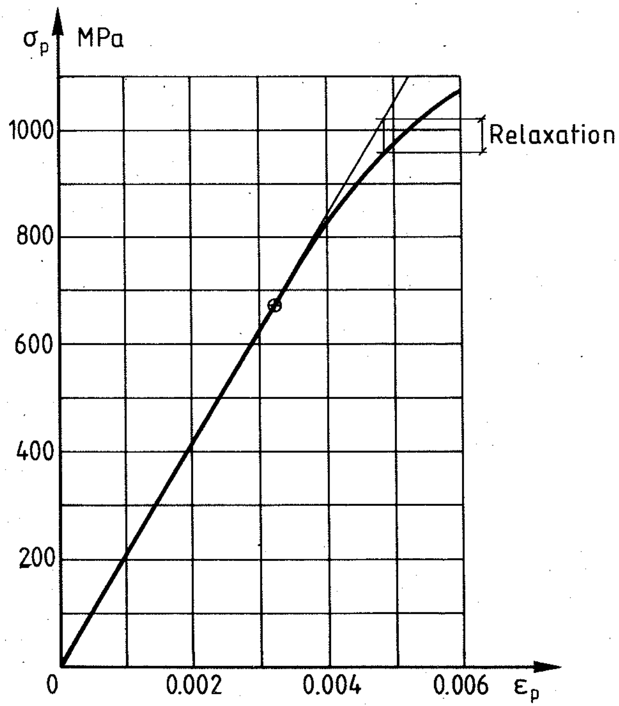

FIG. 4. Stress-Strain Curve for Sustained Strain

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