The Effect of Stirrup Spacing on the Ultimate Load of Reinforced Concrete Beams Subjected to Pure Torsion

John Sander Nielsen

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THE ULTIMATE LOAD OF REINFORCED

CONCRETE BEAMS SUBJECTED TO PURE

TORSION

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PREFACE

The purpose of this report is to clarify the effect of the
stirrup spacing on the load-carrying capacity of reinforced
concrete beams subjected to pure torsion. This is done
both theoretically and by means of a description of 12 tests
carried out at the Department of Structural Engineering of the
Technical University of Denmark in the autumn of 1977.   The
theoretical study and the tests were carried out by the author
in cooperation with Professor dr.techn. Troels Brøndum-Nielsen.
The Danish Council for Scientific and Industrial Research support-
ed the project financially, and the Department of Structural
Engineering assisted in the tests.

November 1982
John Sander Nielsen

SUMMARY

There appear to be very few reports on the effect of stir-
rup spacing on the load-carrying capacity of concrete beams
subjected to torsion (or, in general, on the effect of the
design of the reinforcement). Here, only one report, by Denis
Mitchell, Paul Lampert and Michael P. Collins [1] , will be
discussed in detail (in section 1).

In section 2, a theory is advanced which deals with the pro-
blems on different assumptions than those applied in [1],
while section 3 contains a description of the tests carried
out. Finally, in section 4 the theory is compared with the
results of the tests, and section 5 contains a conclusion.

SUMMARY in Danish   (Dansk Resumé)

Hvad der foreligger af rapporter omhandlende bøjleafstandens (el-
ler i det hele taget armeringsudformningens) betydning for vrid-
ningspåvirkede betonbjælkers bæreevne er yderst sparsomt. Her skal
kun én rapport af   Denis Mitchell, Paul Lampert og Michael P.
Collins, [1], omtales nærmere (afsnit 1).

I afsnit 2 søges opstillet en teori, der, ud fra andre forudsætnin-
ger end de i [1] gjorte, belyser problemerne, medens afsnit 3 er
en beskrivelse af de udførte forsøg. Endelig er teori og forsøg
søgt sammenlignet i afsnit 4, og der afsluttes i afsnit 5 med en
konklusion.

NOTATION

T = Torsional moment

T_n = Torsional strength (Lattice analogy method)

P_a = The yield force in a corner reinforcing bar

P_{ah} = The horizontal component of P_a

P_{av} = The vertical component of P_a

P_b = The yield force in a stirrup

$p(x)$ = Load per unit length of the corner reinforcement

n_{xa} = The force in the reinforcement per unit length (in the longitudinal direction)

n_{ya} = The force in the reinforcement per unit length (in the stirrup direction)

$\left. \begin{array}{c} a \\ b \end{array} \right\}$ = The side lengths of the rectangle formed by the longitudinal reinforcement.

ΣA_e = Area of the total reinforcement in the axial direction

A_{co} = Area of the weakest corner reinforcing bar

s = Stirrup spacing

d_{co} = Diameter of the weakest corner reinforcing bar

u = Circumference of the rectangle formed by the corner reinforcing bar

n = The ratio between the total force in the reinforcement in the longitudinal direction and the sum of the force in the corner reinforcement

1. MODEL PROPOSED BY DENIS MITCHELL, PAUL LAMPERT AND MICHAEL P. COLLINS [1]

In [1], only beams of rectangular cross-section, reinforced with longitudinal reinforcement lying along the sides of the cross-section and with closed stirrups at an angle of 90° with the axis of the beam, are considered.
The statical mode of operation of the beams is assumed to be the same as that of a lattice system with N-lattice, in which the reinforcement consists of flanges and verticals, while the concrete acts in the same way as the diagonals (lattice analogy method - see, for example, [2]).

The problem of finding a maximum permissible stirrup spacing is approached by regarding the corner reinforcing bars as continuous beams supported by the stirrups. The materials are taken to be plastic, and the following assumptions are applied:

1. Same reinforcing force in the direction of the stirrups and the axial direction per unit length (of axial direction and of direction of stirrups).

2. The load on the beams (corner reinforcement) is the diagonal pressure exerted by the concrete.

3. The pressure is assumed to be uniformly distributed, although regarding the loads it is stated that these are of a discrete nature and that they occur randomly due to the impredictable crack location. The result of this is a rigid block type of loading that can substantially reduce the moment. It is further assumed that it is possible for shear force to be transmitted along the tensile cracks. Both these factors are taken into account by putting the maximum moments (in the corner reinforcing bars) at β times the moment in the case of uniformly distributed load.

4. In the calculation of the maximum permissible
   moment, this is assumed to be equal to α
   times the plastic bending moment (without normal
   force) - where α denotes the incremental effect
   of the surrounding concrete on the resistance
   moment.

Equating the permissible with the actual moment, and
comparing this with the available test results, we get:

$$\frac{\Sigma\, A_e}{A_{co}} \cdot \frac{s}{d_{co}} \cdot \frac{s}{u} \le \sqrt{2}\,\frac{\alpha}{\beta} = 25$$

where

$\Sigma\, A_e$ = area of the total reinforcement in the
axial direction

A_{co} = area of the weakest corner reinforcing
bar

s = stirrup spacing

d_{co} = diameter of the weakest corner reinforcing
bar

u = circumference of the rectangle formed by
the corner reinforcing bars.

By plotting the test results as a function of s/d_{co},
it is, in fact, shown that yielding of the reinforcement
occurs when

$$s/d_{co} \le 16$$

From the test material it is further found that

s/u should be chosen smaller than or equal
to 1/8 and the authors advise choosing

$$s \leq 0.3 \text{ m}$$

However, the determination of $\frac{\alpha}{\beta}$ on the basis of the tests reported cannot be said to be reasonably accurate, and on the face of it, the value of $\frac{\alpha}{\beta}$ ($\frac{\alpha}{\beta} = 17.7$) appears high. The latter comment applies particularly in the case of normally reinforced beams, where the plastic moment of resistance of the corner reinforcing bars approaches zero (on account of the normal force) when the torsional moment approaches the ultimate value.

Finally, it must be mentioned that the ratio $\frac{\alpha}{\beta}$ need not necessarily be a constant that is independent of such parameters as the diameter of the corner bars, the stiffness (strength) of the concrete and the design of the reinforcement.

2.   METHOD OF ANALYSIS

Here, the problem is approached on the basis of other
assumptions than those made in [1] with regard to the
mode of operation of the corner reinforcing bars, but
with the same assumptions regarding the construction and
mode of operation of the beam.

2.1   Distribution of pressure from the concrete

Unlike the case in Section 1, the longitudinal reinforce-
ment is here regarded as being flexurally weak, i.e. it
is assumed to have a moment of resistance $\simeq 0$. The
closer the load in the beam to the ultimate load, the
better this assumption will be fulfilled, and for nor-
mally reinforced beams, it will be satisfied when the
ultimate load is reached.

We will now perform the idealization indicated in fig. 1.
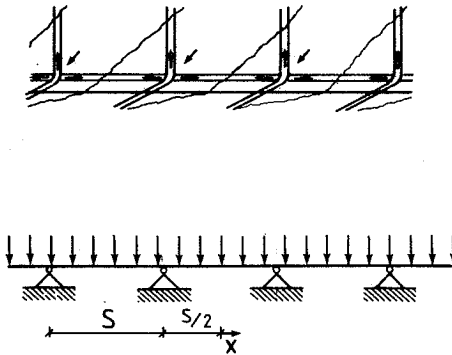
Fig. 1.

Here, the reinforcing bar in the corner is regarded as a
flexurally weak, continous cable, supported at each stirrup
and subjected to a linear load p(x) . Two limit cases
can be imagined for the distribution of p(x):

   a) uniformly distributed load

   b) distribution corresponding to the concrete being
      a homogeneous, perfectly plastic material.

a) uniformly distributed load:

   p(x) = constant = p

   The deflection curve will be a parabola (see [5]
   with the equation:

$$y = \frac{4f}{s^2} x^2$$

   for $-\frac{1}{2} s \leq x \leq \frac{1}{2} s$;

   where f is the camber of the parabola and the
   coordinate system is inserted as shown in fig. 2.
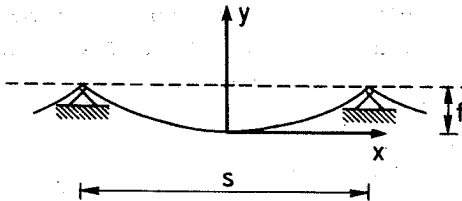
   Fig. 2.

b) elastic model:

$$p(x) = - ky$$

The differential equation for the problem (see [5] is:

$$\frac{d^2y}{dx^2} = - \frac{p(x)}{N}$$

where N is a constant equal to the horizontal force, and k is a constant dependent on the modulus of elasticity of the concrete.

Inserting p(x), we obtain:

$$\frac{d^2y}{dx^2} - \frac{k}{N} y = 0$$

Solution:

$$y = C_1 \cosh \lambda x + C_2 \sinh \lambda x$$

$$\lambda = \sqrt{\frac{k}{N}}$$

C_2 is determined as zero (for reasons of symmetry)

C_1 is determined by integration of $k \cdot y$ over the length s - this integral must be equal to the component of the stirrup forces after the bisector of the corner.

$$C_1 = \frac{\sqrt{2} \cdot P_b \cdot \lambda}{2k \sinh \lambda \frac{s}{2}}$$

where

P_b = the stirrup force.

Inserting C_1 , we obtain:

$$y = \frac{1}{\sqrt{2} \cdot \sqrt{k \cdot N}} \; P_b \; \frac{\cosh \lambda \cdot x}{\sinh \lambda \cdot \frac{s}{2}}$$

$$|p(x)| = \frac{1}{\sqrt{2}} \cdot P_b \; \lambda \; \frac{\cosh \lambda \cdot x}{\sinh \lambda \cdot \frac{s}{2}}$$

where the coordinate system is located as indicated
in fig. 3.
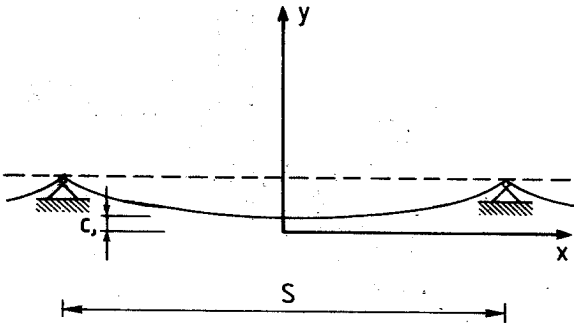
Fig. 3.

It is reasonable to suppose that the distribution
calculated under point b) will be the first to occur,
and that an equalization will take place as the cracking
of the beam becomes more pronounced and as p(x) under
the stirrups exceeds the limit below which the concrete
can be assumed to be perfectly elastic. Numerical values
have not been inserted because several of the parameters
cannot be estimated with reasonable accuracy.

2.2   Bearing capacity formula

In order to arrive at a viable expression for the tor-
sional strength of the beam as a function of the stir-
rup spacing, it is not, however, necessary to know the
true distribution of the concrete pressure along the
reinforcing bars in the corners.

Let us consider a corner member as shown in fig. 4.
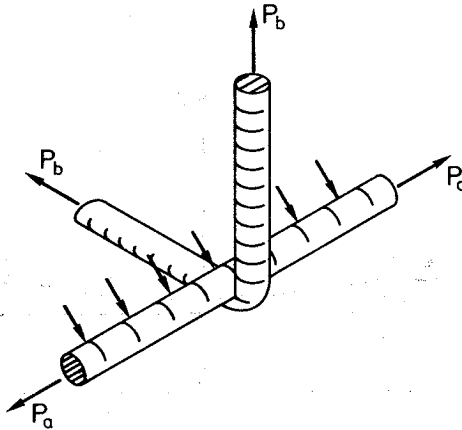
Fig. 4.

Neglecting the part of the concrete stresses that may
possibly be transmitted to the stirrup through direct
contact, we obtain the following by projection on the
plane formed by the bisector of the corner under con-
sideration:
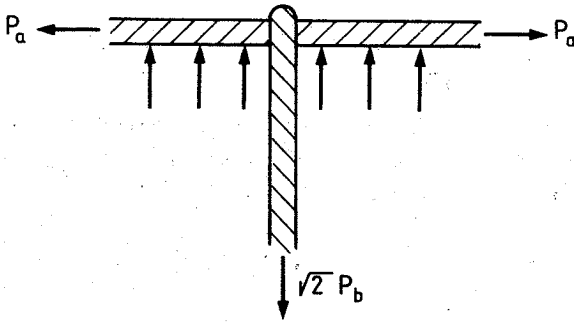
Fig. 5.

As the axial reinforcement is assumed to have no
flexural strength, it is necessary, in order to
create equilibrium, for the axial reinforcing bar
to bend outwards, as shown in fig. 6.
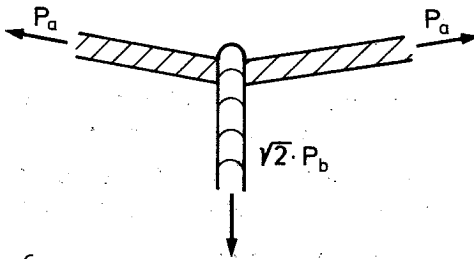
Fig. 6.

with the following force diagram:
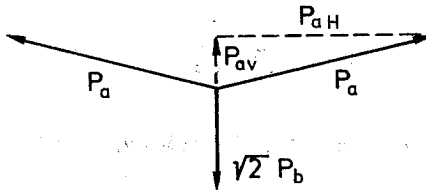
Fig. 7.

From vertical projection, we obtain from fig. 7:

$$P_{av} = \frac{1}{\sqrt{2}} \; P_b$$

Of P_a, only the component P_{aH} remains to resist the torsional moment. The magnitude of P_{aH} is found from the following expression:

$$P_{aH} = \sqrt{P_a^2 - P_{av}^2} = \sqrt{P_a^2 - \frac{1}{2} P_b^2}$$

If we calculate the carrying capacity by means of the lattice analogy method (see, for example, [2]), we obtain the following for underreinforced beams with only one longitudinal bar in each corner:

$$T = 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_{aH}}{\frac{a+b}{2}}} \tag{I}$$

where

  a = the length of one side of the rectangle formed
      by the axial reinforcement,

  b = the length of the other side of the rectangle
      formed by the axial reinforcement,

  s = the stirrup spacing.

The expression normally used for the torsional strength T_n is (see, for example, [2]):

$$T_n = 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_a}{\frac{a+b}{2}}} \tag{II}$$

Inserting T_n and P_{aH} in the expression for T, we obtain:

$$T = T_n \cdot \sqrt[4]{1 - \frac{1}{2}\left(\frac{P_b}{P_a}\right)^2} \tag{III}$$

Fig. 8 shows the expression (the reduction factor):

$$f\left(\frac{P_b}{P_a}\right) = \frac{T}{T_n} = \sqrt[4]{1 - \frac{1}{2}\left(\frac{P_b}{P_a}\right)^2}$$
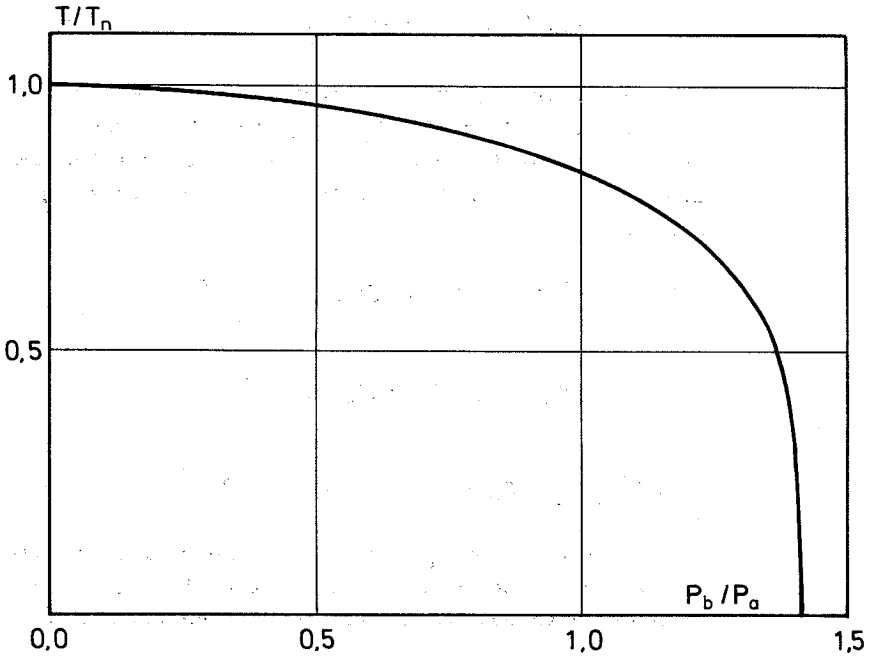
depicted as a function of $\dfrac{P_b}{P_a}$.

Fig. 8.

In accordance with the expression:

$$T = 2ab \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_{aH}}{\frac{a+b}{2}}}$$

$$= 2ab \cdot \sqrt{\frac{P_b}{s} \cdot \frac{2}{a+b}} \cdot \sqrt{P_a^2 - \frac{1}{2} P_b^2}$$

$$= 2ab \cdot \sqrt{\frac{2}{s(a+b)}} \cdot \sqrt{P_a^2 P_b^2 - \frac{1}{2} P_b^4} \qquad \text{(IV)}$$

T increases throughout with increasing P_a, while $T = 0$ for $P_b = 0$ and for $P_b = P_a \cdot \sqrt{2}$. T assumes positive values for $0 \leq P_b \leq P_a \cdot \sqrt{2}$, and must therefore have a maximum for a value of P_b between these bounds.

T is maximum when $P_a^2 P_b^2 - \frac{1}{2} P_b^4$ has a maximum.

$$\frac{\partial}{\partial P_a} (P_a^2 P_b^2 - \frac{1}{2} P_b^4) = 2 P_a^2 P_b - \frac{1}{2} \cdot 4 P_b^3$$

$$2 P_a^2 P_b - \frac{1}{2} \cdot 4 P_b^3 = 0 \qquad \text{for } P_b = P_a$$

In other words, for $P_b > P_a$ we get the maximum torsional strength when the force in one stirrup leg does not reach yield value, but is limited to the value P_a.

In such case, for $P_b \geq P_a$, we obtain from IV :

$$T = 2ab\sqrt{\frac{2}{s(a+b)}}\sqrt{P_a^4 - \frac{1}{2}P_a^4}$$

$$= 2ab\sqrt{\frac{2}{s(a+b)}}\ P_a^2 \cdot \sqrt{\frac{1}{2}}$$

$$= 2ab\sqrt{\frac{2}{s(a+b)}}\ P_a\frac{P_a}{P_b} \cdot P_b \cdot \sqrt{\frac{1}{2}}$$

$$= T_n\sqrt[4]{\frac{1}{2}(\frac{P_a}{P_b})^2} \tag{V}$$

The torsional strength can therefore be determined as

$$T = T_n \cdot f(\frac{P_b}{P_a})$$

where

T_n = the carrying capacity calculated on the basis of the lattice analogy method,

$$f(\frac{P_b}{P_a}) = \begin{cases} \sqrt[4]{1 - \frac{1}{2}(\frac{P_b}{P_a})^2} & \text{for } P_b \le P_a \\ \sqrt[4]{\frac{1}{2}(\frac{P_a}{P_b})^2} & \text{for } P_b \ge P_a \end{cases}$$

The function f is depicted in fig. 9.
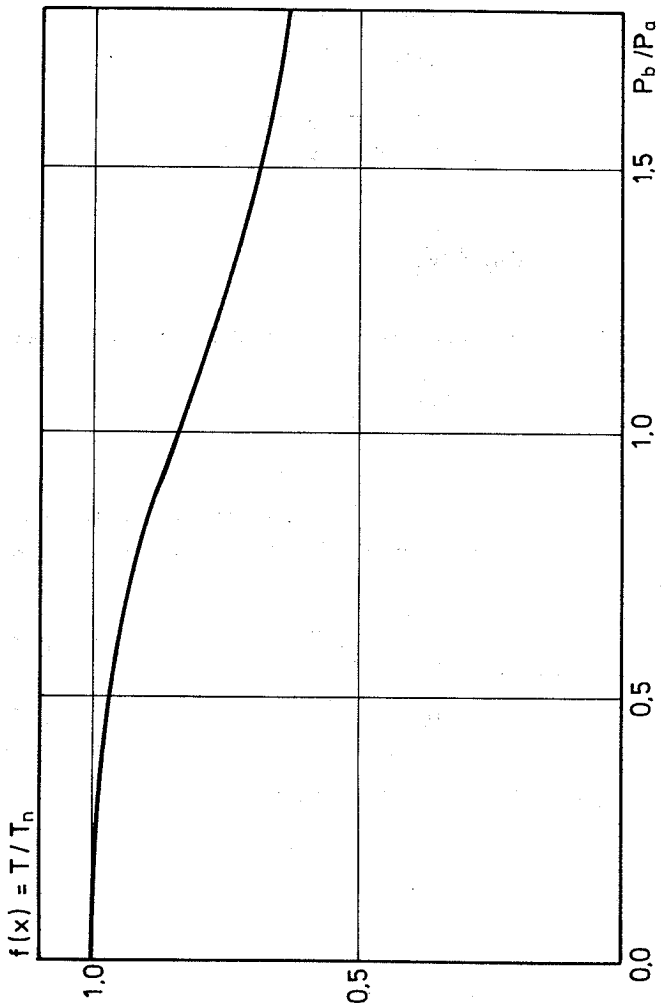
Fig. 9.

Beams with more than four longitudinal reinforcing bars
are calculated in a similar manner. It is assumed here
that failure occurs in the axial reinforcement near the
corners.

Denoting as n the ratio between the total force in the
longitudinal reinforcement and the force in the corner
reinforcement, we can change equation I to:

$$T = 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_{aH} + (n-1)P_a}{\frac{a+b}{2}}}$$

and equation II to:

$$T_n = 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_a}{\frac{a+b}{2}} \cdot n}$$

whereby (without altering the expression for P_{aH}),
equation III becomes:

$$T = T_n \cdot \sqrt{\frac{(n-1) + \sqrt{1 - \frac{1}{2}(\frac{P_b}{P_a})^2}}{n}}$$

Defining γ as the value of the ratio $\frac{P_b}{P_a}$ that maximizes
the expression:

$$T = 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{P_{aH} + (n-1)P_a}{\frac{a+b}{2}}}$$

we obtain, by differentiating with respect to P_b:

$$\gamma = \sqrt{1 - \frac{1}{4}(n-1)^2 + \frac{1}{4}(n-1)\sqrt{(n-1)^2 + 8}}$$

By inserting this expression in

$$T = 2ab\sqrt{\frac{P_b}{s} \cdot \frac{P_{aH} + (n-1)P_a}{\frac{a+b}{2}}}$$

we obtain, in analogy with equation V:

$$T = T_n \cdot \sqrt{\frac{\gamma(n-1) + \sqrt{\gamma^2 - \frac{1}{2}\gamma^4}}{n}} \quad \frac{P_a}{P_b}$$

The torsional strength can therefore be determined for $n \geq 1$ by means of the following expression:

$$T = T_n \cdot f\left(\frac{P_b}{P_a}\right)$$

where T_n = the carrying capacity calculated on the basis of the lattice analogy method

$$= 2 \cdot a \cdot b \cdot \sqrt{\frac{P_b}{s} \cdot \frac{n \cdot P_a}{(a+b)\,2}}$$

$$f\left(\frac{P_b}{P_a}\right) = \begin{cases} \sqrt{\dfrac{(n-1) + \sqrt{1 - \dfrac{1}{2}\left(\dfrac{P_b}{P_a}\right)^2}}{n}} & \text{for} \quad P_b \leq \gamma P_a \\[6mm] \sqrt{\dfrac{\gamma(n-1) + \sqrt{\gamma^2 - \dfrac{1}{2}\gamma^4}}{n}} \left(\dfrac{P_a}{P_b}\right) & \text{for} \quad P_b \geq \gamma P_a \end{cases}$$

where $1 \leq \gamma < \sqrt{2}$ is determined from:

$$\gamma = \sqrt{1 - \frac{1}{4}(n-1)^2 + \frac{1}{4}(n-1)\sqrt{(n-1)^2 + 8}}$$

3. DESCRIPTION OF TESTS

3.1 Object of test and description of test specimens

In order to evaluate the theory formulated in section 2
and to investigate the mode of failure in the case of
big stirrup spacing, a series of tests was carried out
on concrete beams subjected to torsion. The series
comprised 12 beams, two of which were partially pre-
stressed. The arrangement of the reinforcement is shown
in fig. 10 and fig. 11. The tests were performed 14
days after casting of the beams (4 days' curing under
wet sacks and 10 days in the testing laboratory at
about $21^{\circ}C$ and $50^{\circ}RH$).

3.2 General remarks

In the following, a complete description is not given
of the test apparatus, the material properties of the
components, etc., because these were precisely as
described for the test series in [3], to which readers
are referred for additional information.

3.3 Reinforcement

The reinforcement used was round bars and Danish de-
formed bars. The dimensions 7 and 8 mm were normalized
at $860^{\circ}-880^{\circ}C$, followed by air-cooling. One test bar
of each length was extracted and tested, but as the de-
viations in the test results within each dimension
were less than 1%, only the average values of the yield
stress and ultimate strength are given (see table 1).
Figure 12 shows a typical stress-strain curve for the
types of steel used.

Beam No.	Cross Sections 0,3 × 0,3 m	long steel		Stirrups	
I-1	0,25 / 0,25	KS 42 S d = 10 mm	R 37 d = 7 mm s = 81 mm		
I-2	0,250	KS 42 S d = 10 mm	KS 42 S d = 8 mm s = 131 mm		
I-3	0,25 / 0,25	KS 42 S d = 10 mm	KS 42 S d = 10 mm s = 250 mm		
I-4	0,25 / 0,25	KS 42 S d = 10 mm	KS 42 S d = 12 mm s = 350 mm		
I-5	0,25 / 0,25	KS 56 S d = 10 mm	KS 56 S d = 10 mm s = 250 mm		
I-6	0,25 / 0,25	KS 56 S d = 10 mm	KS 56 S d = 12 mm s = 376 mm		

Fig. 10.
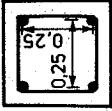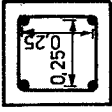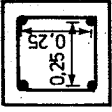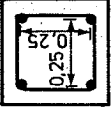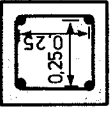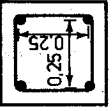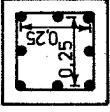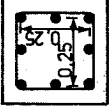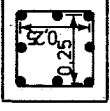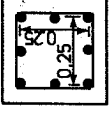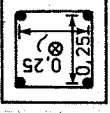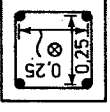
Beam No.	III-1	III-2	III-3	III-4	III-5	III-6
Cross Sections 0,3 × 0,3 m						
long steel	KS 42 S d = 10 mm	KS 42 S d = 10 mm	KS 42 S d = 10 mm	KS 42 S d = 10 mm	KS 42 S d = 10 mm *	KS 42 S d = 10 mm *
Stirrups	KS 42 S d = 8 mm $s = \frac{131}{2}$ mm	KS 42 S d = 10 mm s = 125 mm	KS 42 S d = 12 mm s = 175 mm	KS 90 S d = 12 mm s = 339 mm	KS 42 S d = 10 mm s = 125 mm	KS 42 S d = 12 mm s = 175 mm

* and a ½" prestressed tendon placed in the centre of the cross section.

Fig. 11.

Kvalitet	Dimension mm	P_f KN	ε_f	P_{brud} KN
R 37	7	12.50	0.00140	18.0
KS 42 S	8	20.40	0.00200	31.1
	10	38.57	0.00226	50.9
	12	53.96	0.00202	73.1
KS 56 S	10	49.75	0.0032	63.2
	12	74.40	0.0032	97.5
KS 90 S	12	104.70	0.00455	113.8

Table 1. Steel strengths
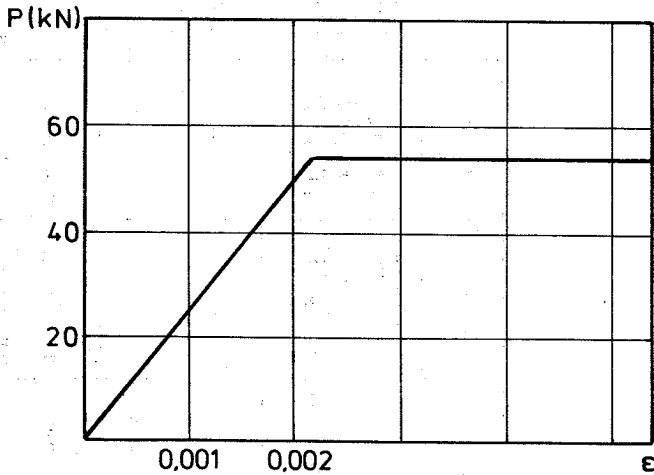
Fig. 12

In beams II-5 and II-6, a half-inch prestressed tendon
(English normal relaxation steel) was also used.

The ultimate load for the cables was 175 kN, and the cables
were prestressed to about 140 kN (about 80%). The curve
for the loss of prestress on account of relaxation has
been taken from tests carried out at the Structural Re-
search Laboratory of the Technical University of Denmark
in 1975 and is published in [6]. The steel tested here
was of the same quality as the steel used for the tests,
except that the tendon diameter was 0.6". The prestress
was 80%.

3.4   Concrete

Three types of concrete were used, with the following
compositions, where the cement used was Portland rapid.

For beams I-1, I-2, I-3, I-4 and I-5:

water	164 kg/m^3
cement	205 kg/m^3
fine agg.	846 kg/m^3
coarse agg.	1119 kg/m^3

For beam I-6:

water	200 kg/m^3
cement	128 kg/m^3
fine agg.	1305 kg/m^3
coarse agg.	945 kg/m^3

For beams II-1, II-2, II-3, II-4, II-5 and II-6:

water	167 kg/m^3
cement	300 kg/m^3
fine agg.	717 kg/m^3
coarse agg.	1167 kg/m^3

For each beam, 6 cylinders were cast for compression
testing and 6 cylinders for splitting tests.

The strength values measured and their standard devia-
tions are shown in table 2.

Table 2. Concrete strengths

	Compression strength MPa		Splitting strength MPa	
Beam No.	mean value	devia-tion	Mean value	devia-tion
I-1	20.99	0.830	2.662	0.101
I-2	20.52	0.653	2.149	0.356
I-3	22.23	0.643	2.780	0.128
I-4	21.50	1.442	2.343	0.249
I-5	22.08	0.580	2.350	0.413
III-1	38.68	1.345	3.269	0.145
III-2	37.88	1.260	3.326	0.165
III-3	38.16	1.150	3.579	0.213
III-4	39.62	1.112	3.204	0.284
III-5	44.16	0.643	3.596	0.159
III-6	44.80	2.093	3.239	0.239

3.5   Test arrangement

Fig. 15 shows the statical mode of operation of the
test arrangement, while fig. 16 shows a photo of the
apparatus.
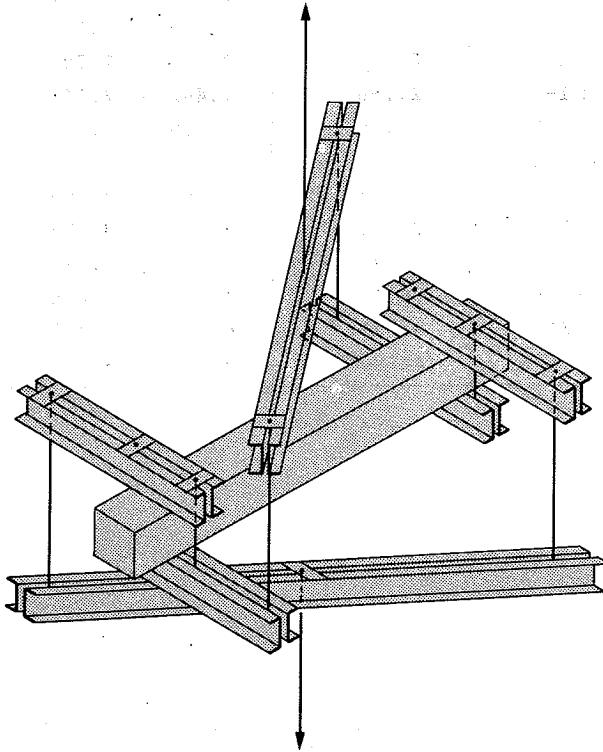
Fig. 15
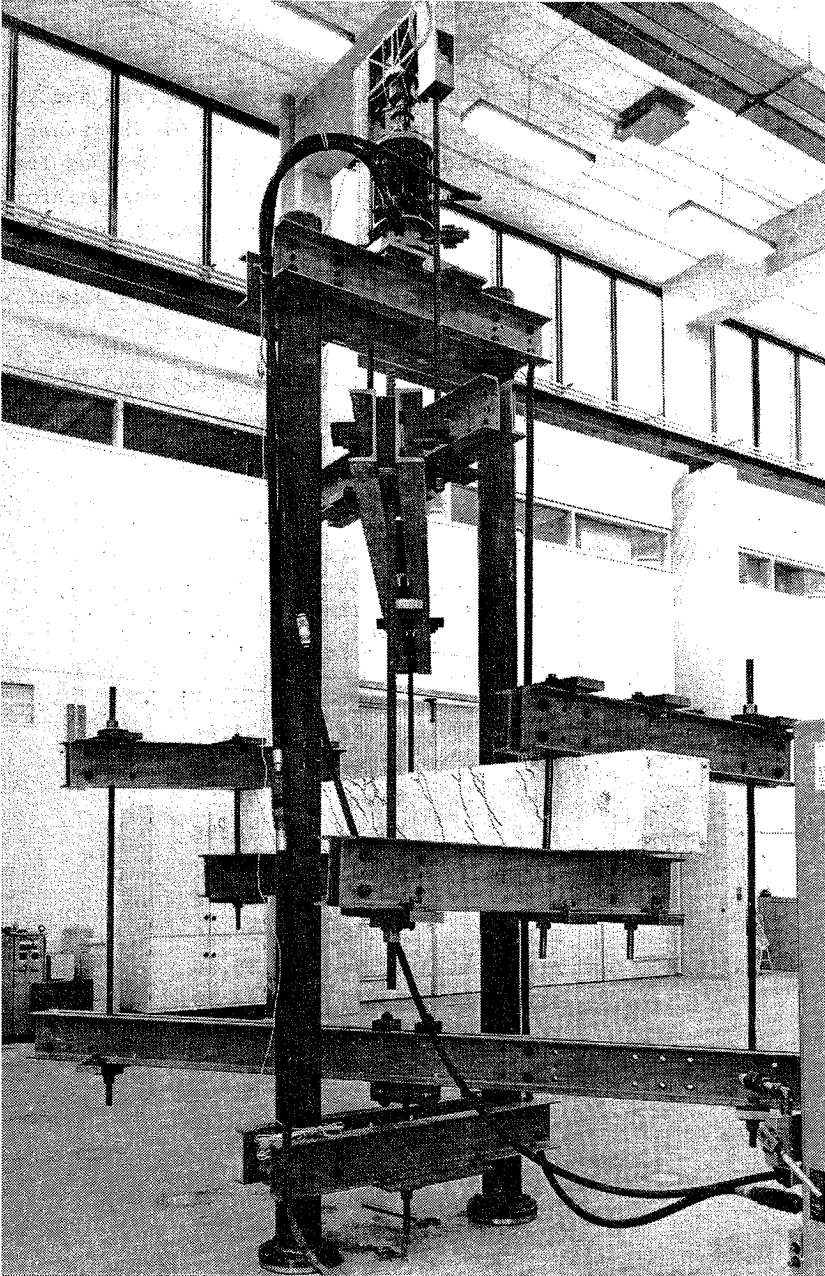
Fig. 16.

3.6   Measurements

In the tests, the load at which cracks occurred and the
ultimate load were registered (see table 3), together
with maximum crack widths occurring at the various loads
(see fig. 17). In addition, the strain in the reinforce-
ment was measured by means of strain gauges placed on
two of the longitudinal reinforcing bars and on the stir-
rups on either side of the middle stirrup (see appendix).

The strain measurements in the prestressed tendons were
taken by means of strain gauges placed as shown in fig.
18.

Table 3. Cracking load and ultimate load

Beam No.	Cracking load kNm	Ultimate load kNm
I-1	18.91	22.22
I-2	17.84	21.04
I-3	17.84	17.84
I-4	17.84	17.84
I-5	15.72	18.86
III-1	25.31	41.72
III-2	23.18	35.96
III-3	22.54	35.96
III-4	23.18	23.18
III-5	30.64	41.29
III-6	27.44	37.03

CRACK WIDTH
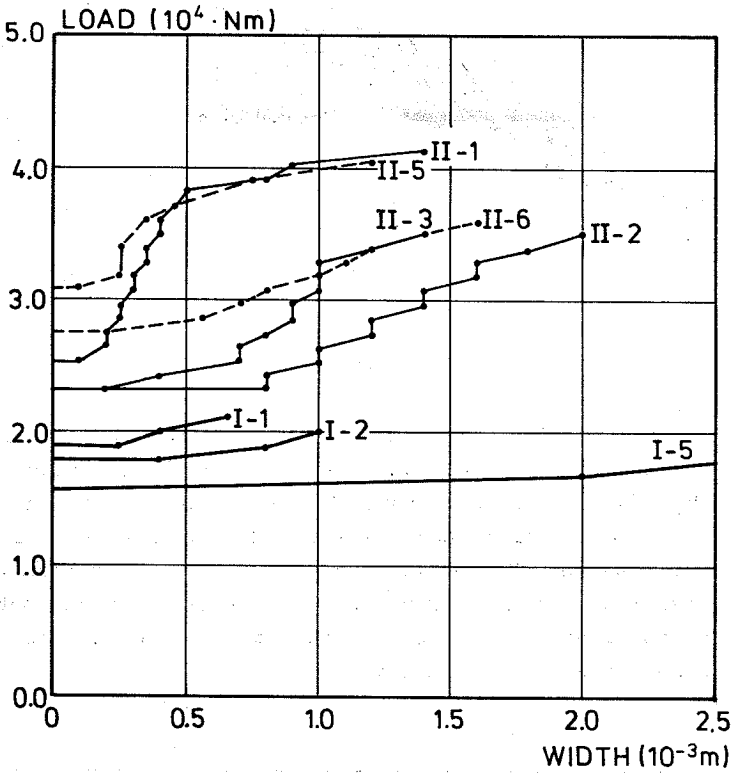
Fig. 17
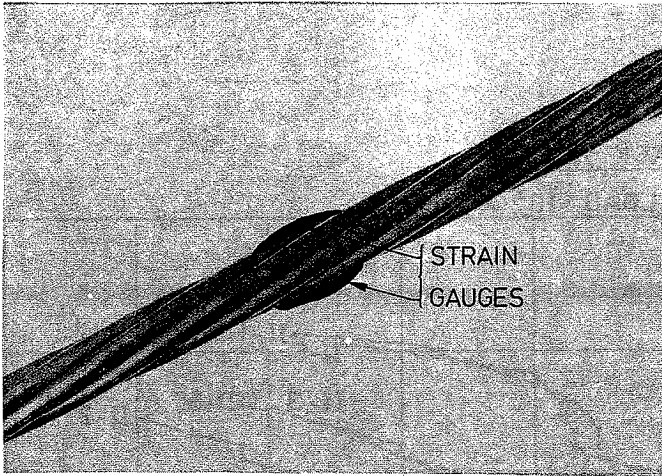
Fig. 18.

On account of the inclination of the wires, the strain
measure is not equal to the elongation of the tendon.
Therefore, on a test bar, the relationship between
the strain measure used here and the force in the tendon
was measured. In addition, the rotation of the beam was
measured with angular potentiometers glued to the bottom
of the beam (see appendix). The positions of the potent-
iometers are shown in fig. 19.

potentiometer
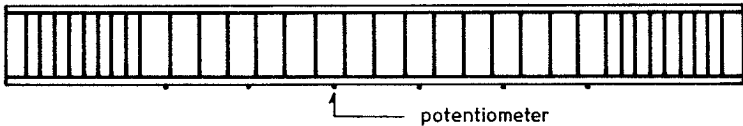
Fig. 19.

3.7   Date-processing

All the test results were put out on punch tape, con-
verted to punch cards and then processed by means of
a computer program developed for the purpose. The
results from the material tests were coded on punch
cards and also included in the program.

The following were computed for each loading step:

 Rotations
 Mean rotation per unit of length
 Strain in reinforcement
 Mean strain in reinforcement
 Differential strain
 Forces in reinforcement

In addition, the material constants and the standard
deviations on these were computed.

The following curves were plotted (see appendix):

 Loading history
 Rotation
 Stress-strain curve for the beam
 Strain in the reinforcement (mean and differential)
 Forces in the reinforcement.

4. COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL
   RESULTS

The comparison between test and theory is mainly con-
centrated on the following two points:

1. comparison between the modes of behaviour assumed
   in sections 1 and 2 and those observed in the
   tests;

2. comparison between the calculated and the mea-
   sured values of the ultimate strengths of the
   beams.

Re. 1.
   The theory of D. Mitchell, P. Lampert and M. P.
   Collins, described in section 1, operates with a
   deflection pattern for the corner reinforcing bars
   as shown in fig. 20, whereas the theory in section
   2 assumes that the deflection pattern is a collec-
   tion of smooth curves composed of parabolic and
   hyperbolic cosine curves.

corner reinforcing bar

stirrup   stirrup   stirrup
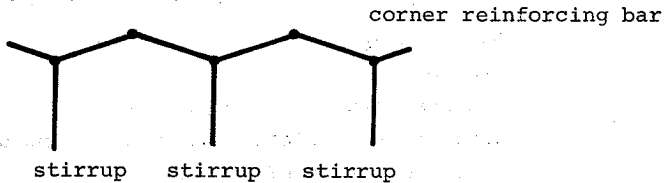
Fig. 20.

If we consider a beam in which the reinforcement
has reached its yield stress (normally reinforced),
the deflection patterns for the corner reinforcing
bars at the ultimate load will not change during
unloading, and we can therefore obtain an impres-
sion of the mechanism chosen by the reinforcing
bar by cutting away the cover.

Fig. 21.

The photo shown in fig. 21 is taken from P. Lampert,
P. Lüchinger and B. Thürlimann: Torsionsversuche an
Stahl- und Spannbetonbalken [4] and shows the beam
T_9, which contains not only the reinforcement shown,
but also a prestressed tendon in the middle of the
cross-section. It will be seen that the deflection
pattern is a collection of parabolic curves with
points of discontinuity under the stirrups, and
there is nothing to indicate the formation of a
plastic hinge halfway between 2 stirrups.

Fig. 22 and fig. 23 are photos of corner reinforcing
bars cut out of beams II-5 and II-6, and the same
tendency can be noted here.
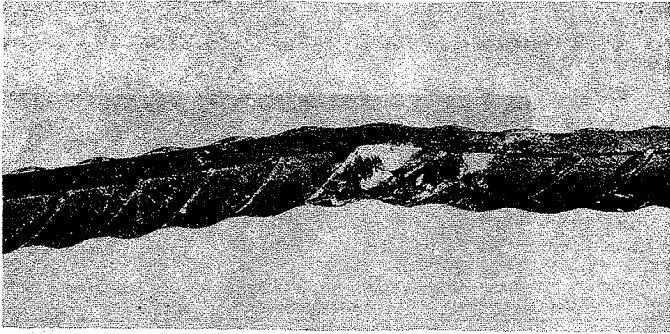
Fig. 22.
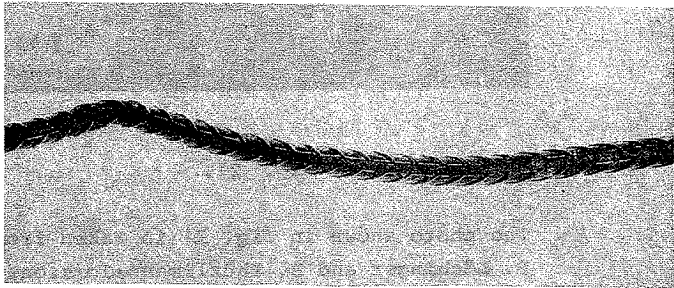
Fig. 23.

In addition, it is noted in fig. 23 that the two
curves meet rather sharply at the point of dis-
continuity, which might indicate that the assumption
made in section 2.2 - that no compressive stresses
in the concrete are transmitted directly to the
stirrup through contact pressure - is a reasonable
one.

With regard to the failure mechanism of the beam,
it is noted that no yielding of the reinforcement
took place at stirrup spacings greater than the
distance between the longitudinal bars in the cor-
ners. A failure mechanism occurred that was

analogous to that occurring for unreinforced beams
- flexural failure about an axis parallel with the
beam axis and located on one side (see fig. 24).

axis of rotation
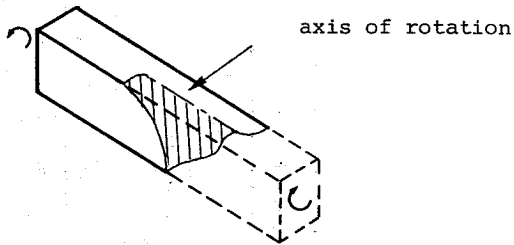
Fig. 24.

This failure mechanism does not activize any of
the stirrups of the longitudinal bars, and if the
ultimate load is greater than the cracking load,
it must presumably be due solely to dowel action.

Re. 2.

In addition to the authors' own tests, described
in section 3, the comparison between test and
theory will also take account of 6 tests carried
out by D. Mitchell, P. Lampert and M. P. Collins,
described in [1].

Fig. 25 shows the test specifications, and table
4 shows all theoretical and experimental bearing
capacities.

Fig. 25.

Table 4.

Beam No.[ ]	T_{exp} i.k.	T_n i.k.	$\dfrac{T_{exp}}{T_n}$	T_{theory} i.k.	$\dfrac{T_{exp}}{T_{theory}}$
C 0	441	584	0.76	356	1.24
C 1	522	594	0.88	443	1.18
C 2	590	594	0.99	575	1.03
C 6	466	546	0.85	491	0.95
C 7[+]	450	594	0.76	465	0.97
C 8	294	367	0.80	302	0.97
Beam No.	KNm·10	KNm·10		KNm·10	
I-1	2.222	1.929	1.15	1.910	1.16
I-2	2.104	1.929	1.09	1.862	1.13
I-3	1.784	1.929	0.92	1.624	1.10
I-4	1.784[*]	1.929	–	1.373	–
I-5	1.886	2.487	0.76	2.092	0.90
I-6	[*]	2.487	–	1.949	–
III-1	4.172	3.857	1.08	3.780	1.10
III-2	3.596	3.857	0.93	3.564	1.01
III-3	3.596	3.857	0.93	3.128	1.15
III-4	2.318[*]	3.857	–		–
III-5	4.129	3.868	1.07	3.791[Δ]	1.09
III-6	3.703	3.868	0.96	3.133[Δ]	1.18

[*] cracking load

[Δ] n estimated at 2

The total test material comprises beams with 4, 8 and 16 longitudinal reinforcing bars and beams with 4 longitudinal reinforcing bars plus prestressed tendon.

Fig. 26-29 show the curves for the carrying capacity (in accordance with section 2.2) for beams with these numbers of longitudinal reinforcing bars (n = 1, 2 and 4), together with the results of the tests.

FIG. 26.

$f(x) = T/T_n$

	own tests
	tests from [1]
	n=2

P_b/P_a

1,0

0,5

0,0

1,5

1,0

0,5

0,0
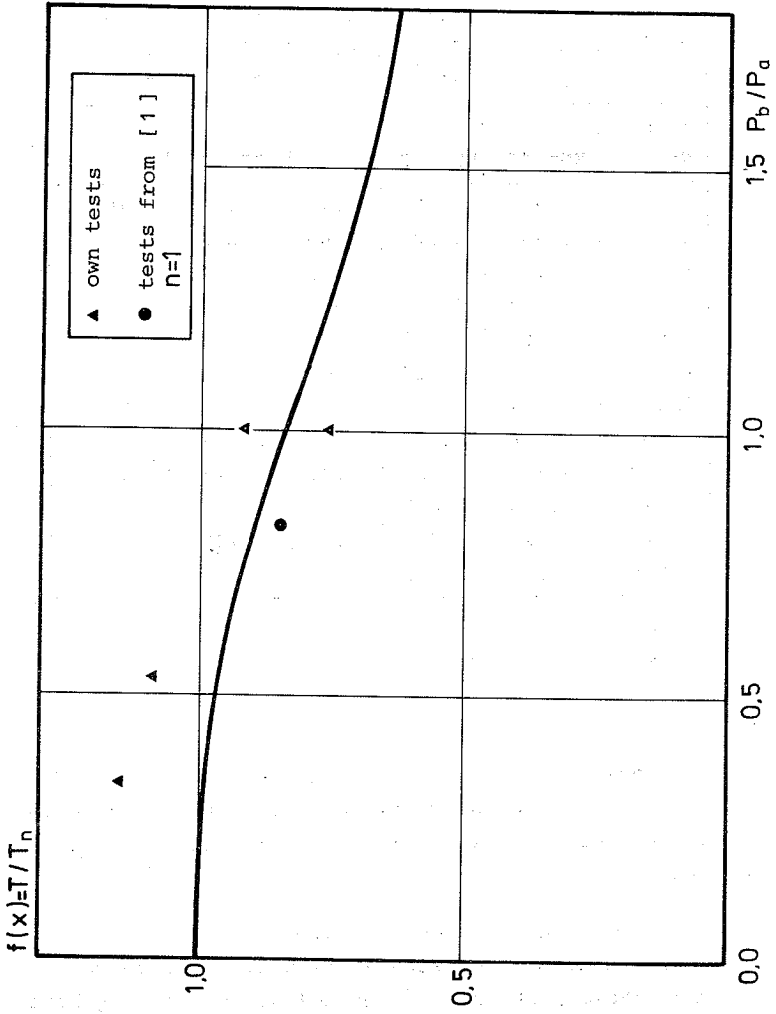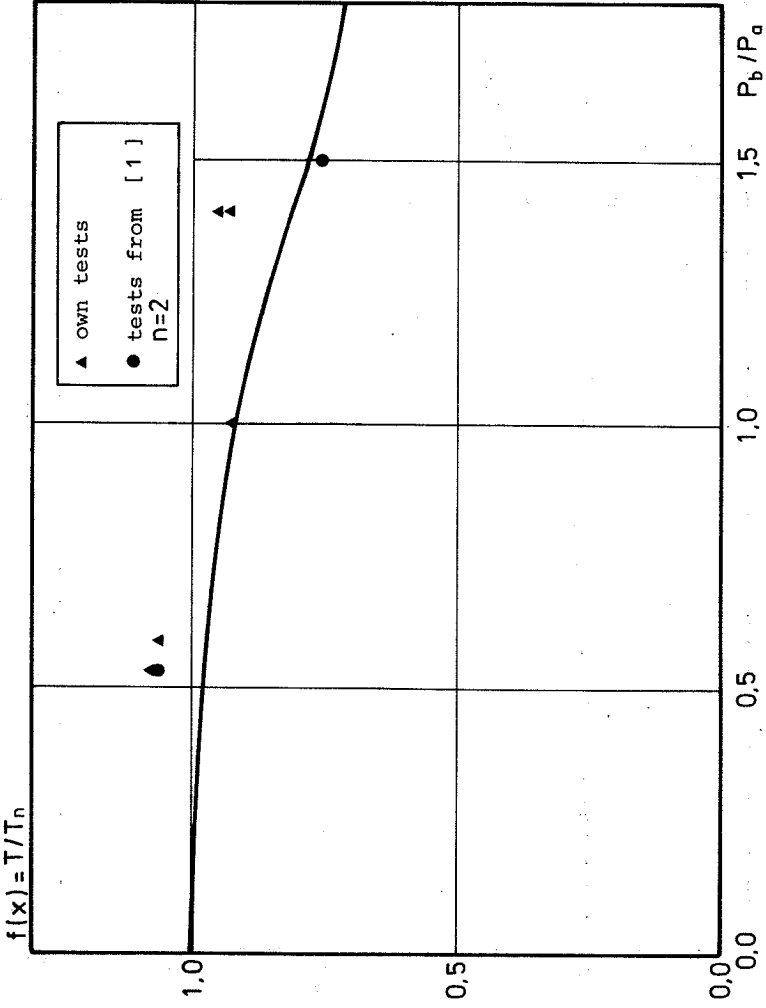
FIG. 27.
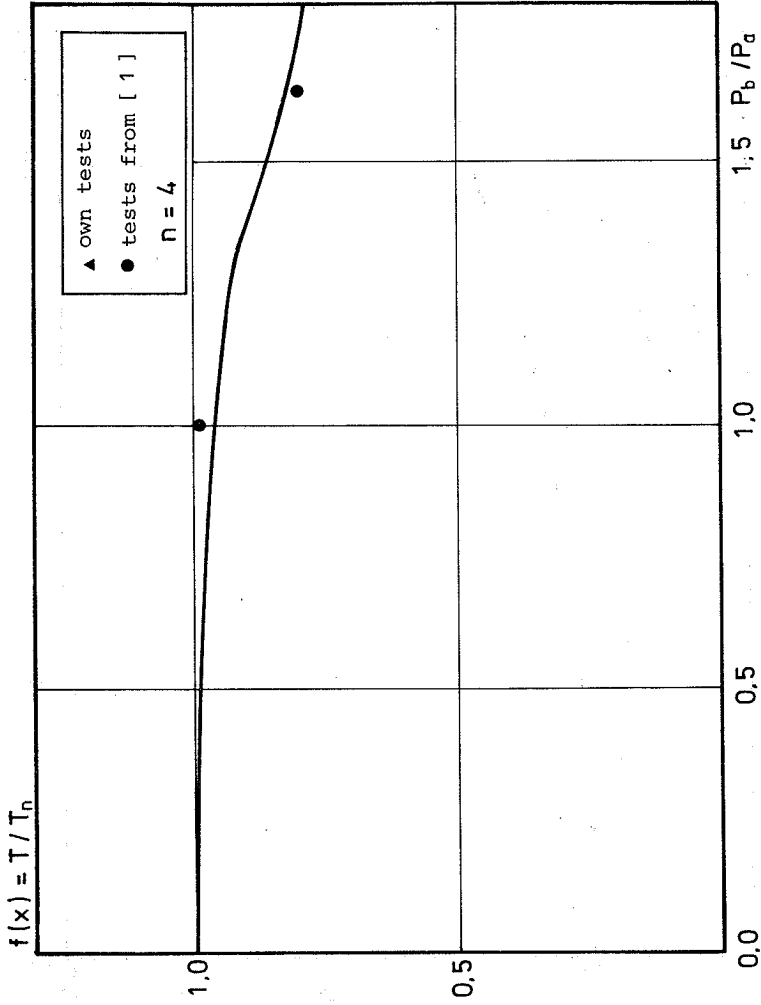
FIG. 28.
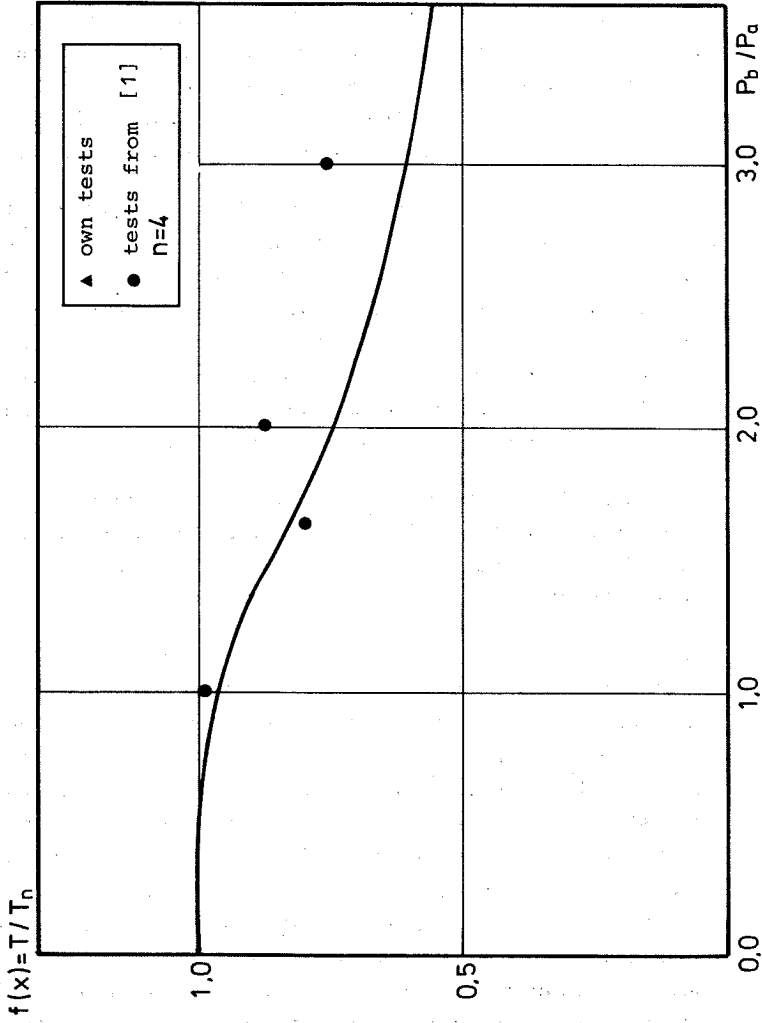
FIG. 29.

5. CONCLUSION

In this report it is demonstrated that, for normally reinforced beams in torsion, the effect of the stirrup spacing can be calculated by considering the interaction between concrete and reinforcement in a corner of a cross section. The correspondence between the observed and the anticipated values of the failure load (fig. 26 to fig. 29) seems to be satisfactory.

In particular, the results of the tests confirm that the carrying capacity of beams with a stirrup spacing that is less than the distance between two corner reinforcing bars decreases with increasing stirrup spacing. In addition, the tests also seem to confirm that the carrying capacity is not only dependent on the stirrup spacing, but also on the character of the reinforcing bars at the corners of the cross section.

The ultimate carrying capacity of the two partially prestressed beams II-5 and II-6 seems to show the same dependence on the stirrup spacing as for the traditionally reinforced beams.

With regard to fig. 29, better correspondence between experiment and theory could be achieved by assuming that only part of the compressive stresses in the concrete was transmitted to the longitudinal reinforcing bar and that the residue was transmitted directly to the stirrups through contact pressure. However, this only applies to the beams in fig. 29 (beams CO and C1), and as, furthermore, the stirrup arrangement is atypical (double and triple stirrups), with a considerably better possibility for direct force transmission than with a normal arrangement, the assumption in section 2.2 will be maintained (that the whole of the compression in the

concrete is transmitted to the longitudinal reinforcing bar in the corner). It is shown that the carrying capacity can be calculated by means of the following expression:

$$T = T_n \cdot f\left(\frac{P_b}{P_a}\right)$$

where,

T_n = the carrying capacity calculated in accordance with the lattice analogy method

$$= 2 \cdot a \cdot b \cdot \sqrt{n_{xa} \cdot n_{ya}}$$

where:

a,b = the side lengths of the rectangle formed by the longitudinal reinforcement,

n_{xa} = the force in the reinforcement per unit length (in the longitudinal direction),

n_{ya} = the force in the reinforcement per unit length (in the stirrup direction)

$$f\left(\frac{P_b}{P_a}\right) = \begin{cases} \sqrt{\dfrac{\left(n-1+\sqrt{1-\frac{1}{2}\left(\frac{P_b}{P_a}\right)^2}\right)}{n}} & ; \quad P_b \leq \gamma \cdot P_a \\[4mm] \sqrt{\gamma \dfrac{\left(n-1+\sqrt{1-\frac{1}{2}\gamma^2}\right)}{n} \dfrac{P_a}{P_b}} & ; \quad P_b \geq \gamma \cdot P_a \end{cases} \quad \text{for}$$

where: $1 \leq \gamma < \sqrt{2}$

$$\gamma = \sqrt{1 - \frac{1}{4}(n-1)^2 + \frac{1}{4}(n-1)\sqrt{(n-1)^2 + 8}}$$

n = the ratio between the total force in the reinforcement in the longitudinal direction and the sum of the force in the corner reinforcement,

P_b = the yield force in a stirrup,

P_a = the yield force in a corner reinforcing bar.

––––––––––

Furthermore, it has been observed that the mode of failure changes at stirrup spacings greater than the distance between two corner bars, to a failure pattern analogous to that for unreinforced beams. Beams with such a failure pattern have a very low ultimate strength and should therefore be avoided.

The theory developed here can be used as follows to formulate simple design rules:

An increase in the ratio $\frac{P_b}{P_a}$ beyond the value γ , where γ is determined from the expression on page 45 , will not result in yielding in the stirrups, and it is therefore appropriate to limit P_b to the value $\gamma \cdot P_a$.

γ assumes only values between 1 and $\sqrt{2}$, and as a γ - value near $\sqrt{2}$ would give big deflections of the corner bars, it is proposed that the value of γ be put at 1 and thus that

$$P_b \leq P_a$$

be prescribed.

For beams with 8 or more longitudinal reinforcing bars of the same dimension (and $n_{xa} = n_{ya}$), this requirement is synonymous with a stirrup spacing of $s \leq \frac{1}{4}(a+b)$. If this latter requirement is extended to apply to all reinforcing bars, we get:

$P_b \leq P_a$

$s \leq \frac{1}{4}(a+b)$

which means that we have limited the reduction in the bearing capacity to maximum 3% (see curves for bearing

capacity, fig. 26 to fig. 29). In addition, it must be checked, with a reasonable degree of certainty, that no crack can form that is not crossed by a stirrup. This requirement can possibly be included by requiring instead that:

$$P_b \leq P_a$$

$$s \leq \frac{1}{2}\min(a,b)$$

APPENDIX

In addition to photos of the crack patterns observed in the
tests, this appendix contains the following six curves for
each beam:

1. Loading histogram

    abscissa: time in minutes from commencement of test

    ordinate: torsional moment

    comments: owing to the fact that the dead load of the
        beam contributes to the load, the curve
        does not start at (0,0).

2. Rotations

    abscissa: mutual placing of the angular potentiome-
        ters (indicated by vertical lines)

    ordinate: angle of rotation in relation to 1st mea-
        suring point

    comments: each corresponds to one loading phase,
        the magnitude of which (in Mpm) is given
        on the right of each curve.

3. Stress-strain diagrams

    abscissa: the rotation between measuring points 2
        and 5

    ordinate: torsional moment.

4. Strains in reinforcement

    abscissa: torsional moment

    ordinate: mean strain from one pair of gauges.

5.   Differential strains

      abscissa: torsional moment

      ordinate: difference between one pair of gauges

6.   Forces in reinforcement

      abscissa: torsional moment

      ordinate: force in one reinforcing bar

      comments: the yield stresses for the various bars
                      are given on the left of the figures.

The sides of the beams are numbered as follows:
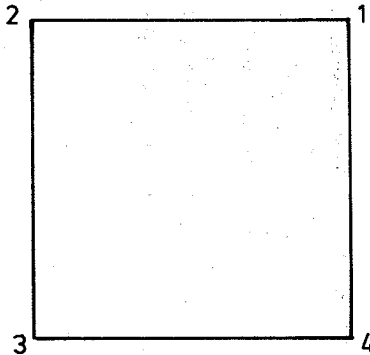
Fig. 30.   Beam seen from end

The numbers of the relevant side face are given on the right
of each photo.

BEAM NO. I-1
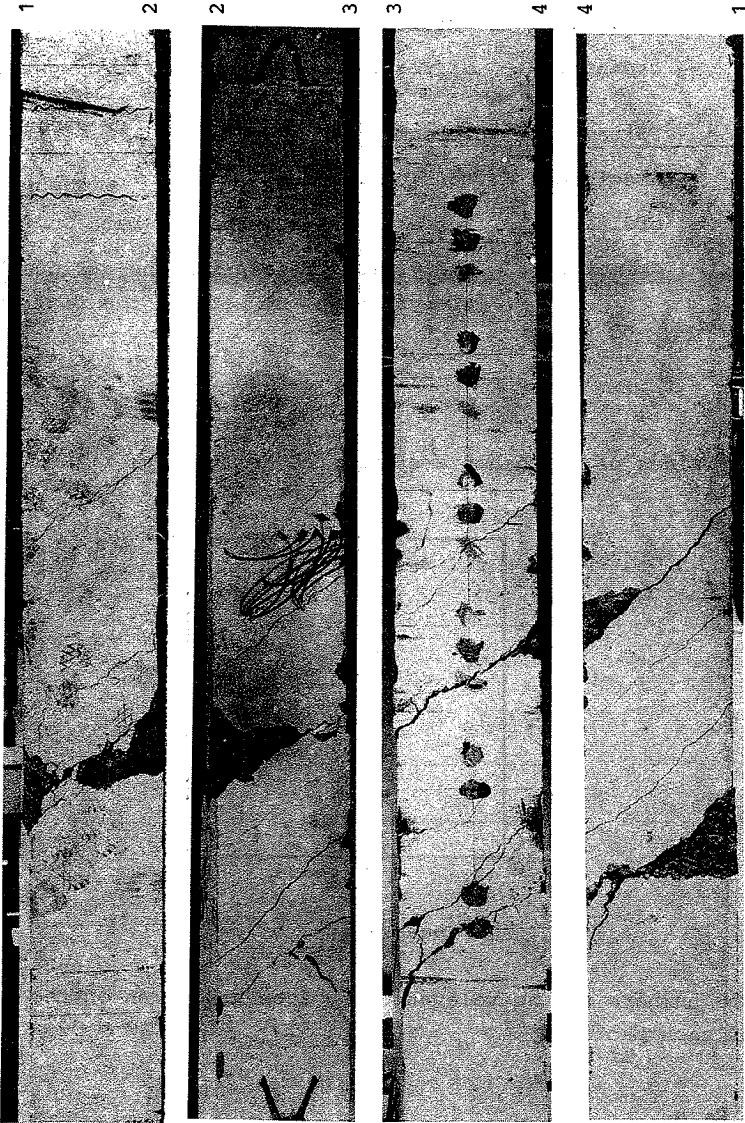
BEAM NO. I-2
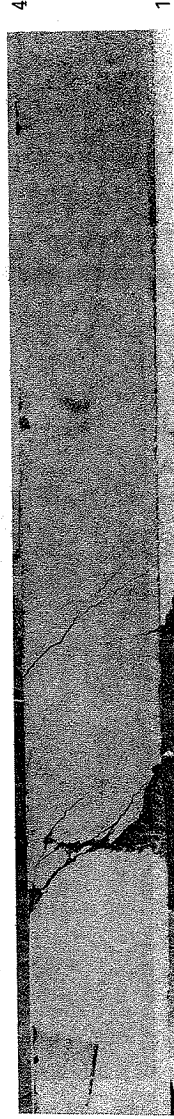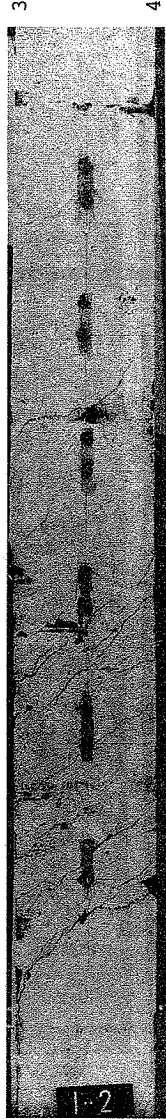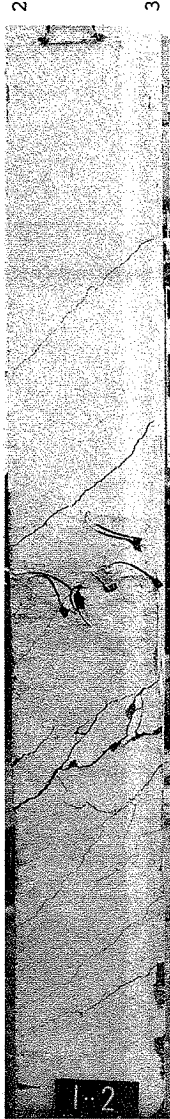
BEAM NO. I-3
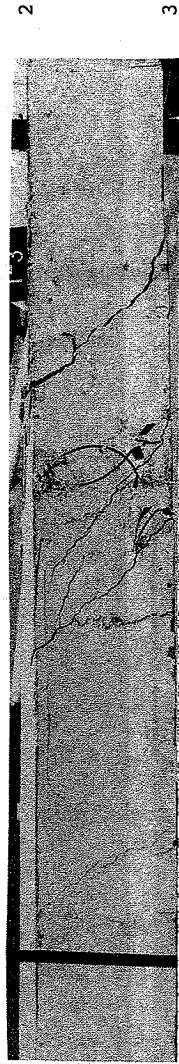
BEAM NO. I-4
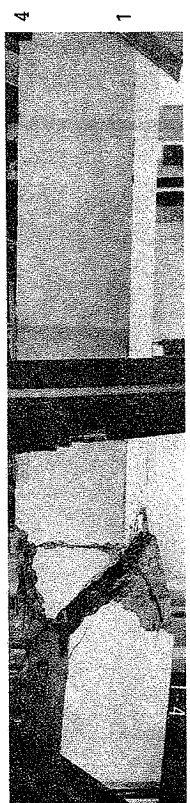
BEAM NO. I-5
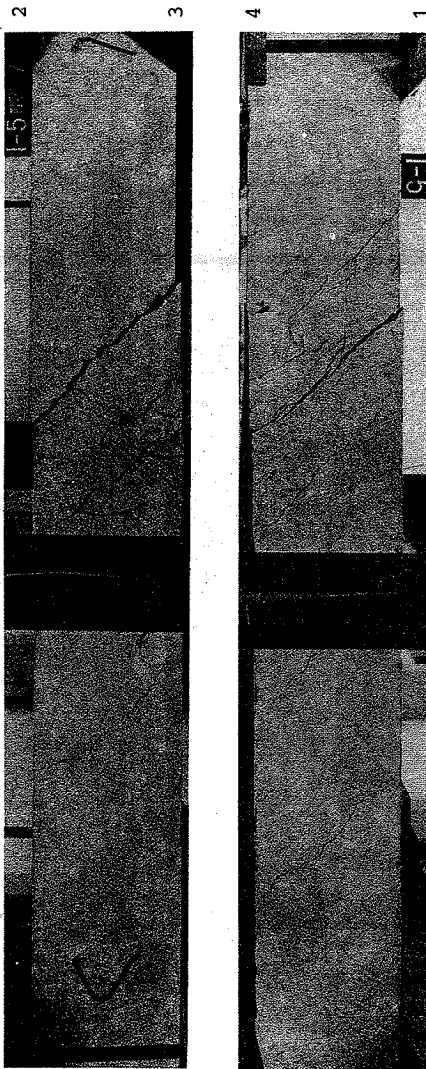
BEAM NO. III-1
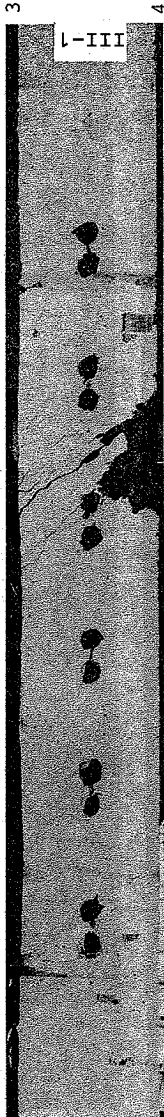
BEAM NO. III-2

BEAM NO. III-3

BEAM NO. III-4

LOADING HISTORY
BEAM NO. I 1

ROTATIONS
BEAM NO. I 1

ROTATIONS

LOAD (MPM)   BEAM NO. I 1

STRAINS (O/OO)   BEAM NO. I 1

AVERAGE GAUGE STRAINS

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (0/00)   BEAM NO. I 1

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. I 1

LOADING HISTORY
BEAM NO. I 2

LOAD (MPM)

5.0

⊙ SCANS

4.0

3.0

2.0

1.0

0.0
0.   40.   80.   120.   160.   200.
TIME (MIN)

ROTATIONS
BEAM NO. I 2

DEG.

9.0   2.14
8.0
7.0
6.0
5.0
4.0
3.0   2.01
2.0   1.89
1.0   1.79
0.0
-1.0
0.   30.   60.   90.   120.   150.
POSITION (CM)

ROTATIONS

LOAD (MPM)   BEAM NO. I 2

ROTATION (DEG/M)

AVERAGE GAUGE STRAINS

STRAINS (0/00)   BEAM NO. I 2

○ STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

LOAD (MPM)

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (0/00)   BEAM NO. I 2

o STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

LOAD (MPM)

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. I 2

o STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

LOAD (MPM)

LOADING HISTORY

LOAD (MPM)   BEAM NO. I 3

ROTATIONS

DEG.   BEAM NO. I 3

ROTATIONS
BEAM NO. I 3

AVERAGE GAUGE STRAINS
BEAM NO. I 3

DIFFERENCE BETWEEN GAUGE STRAINS

STR-INS (0/00)   BEAM NO. I 3

o STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

LOAD (MPM)

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. I 3

o STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

LOAD (MPM)

LOADING HISTORY
BEAM NO. I   4

LOAD   (MPM)

ROTATIONS
BEAM NO. I   4

DEG.

ROTATIONS

LOAD (MPM)   BEAM NO. I 4

AVERAGE GAUGE STRAINS

STRAINS (O/OO)   BEAM NO. I 4

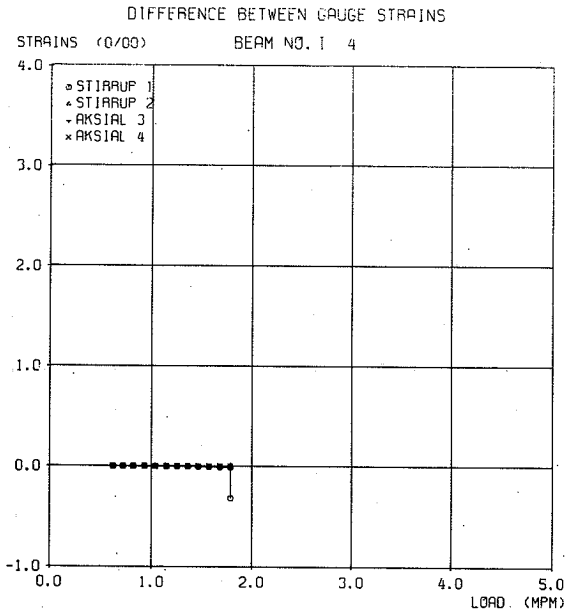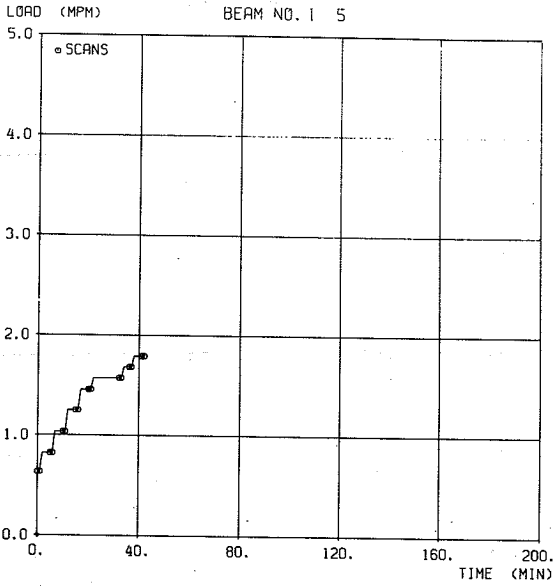DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (0/00)   BEAM NO. I 4

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. I 4

LOADING HISTORY
LOAD (MPM)　　BEAM NO. I　5

⊙ SCANS

TIME (MIN)

ROTATIONS
DEG.　　BEAM NO. I　5

1.79

1.68

1.57

POSITION (CM)

ROTATIONS

LOAD (MPM)   BEAM NO. I 5

AVERAGE GAUGE STRAINS

STRAINS (O/OO)   BEAM NO. I 5

o STIRRUP 1
▲ STIRRUP 2
+ AKSIAL 3
× AKSIAL 4

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (0/00) BEAM NO. 1 5

REINFØRCEMENT FORCES

FØRCES (KP) BEAM NO. 1 5
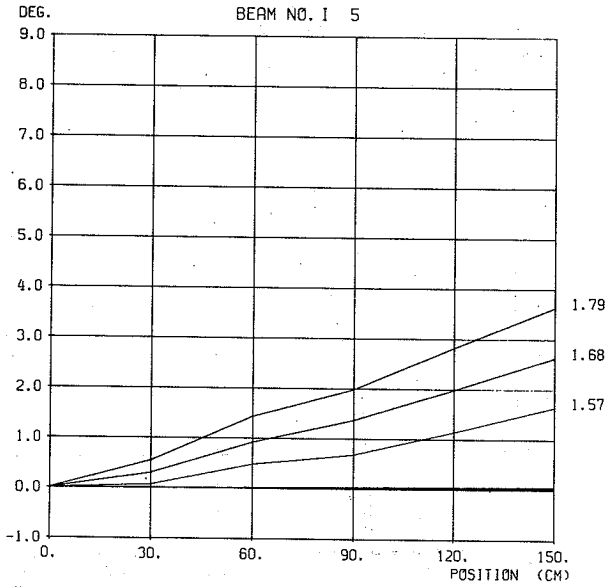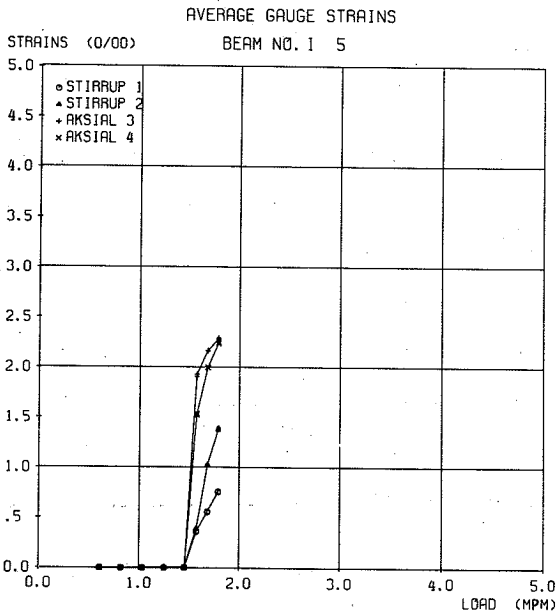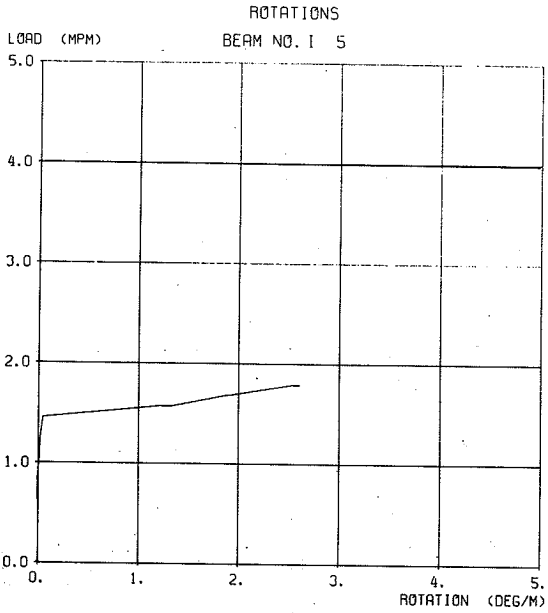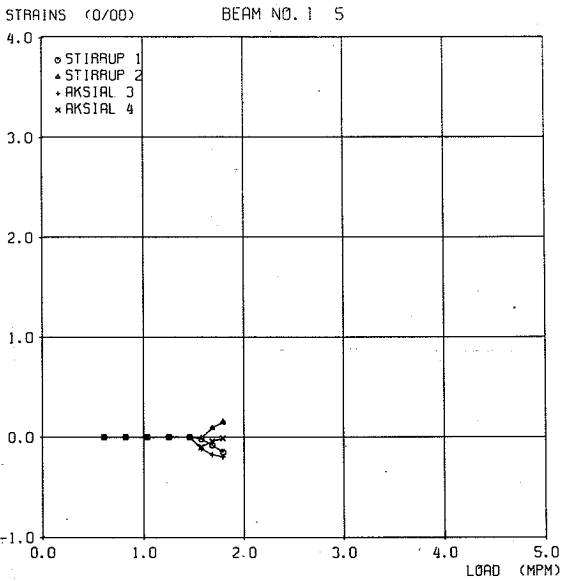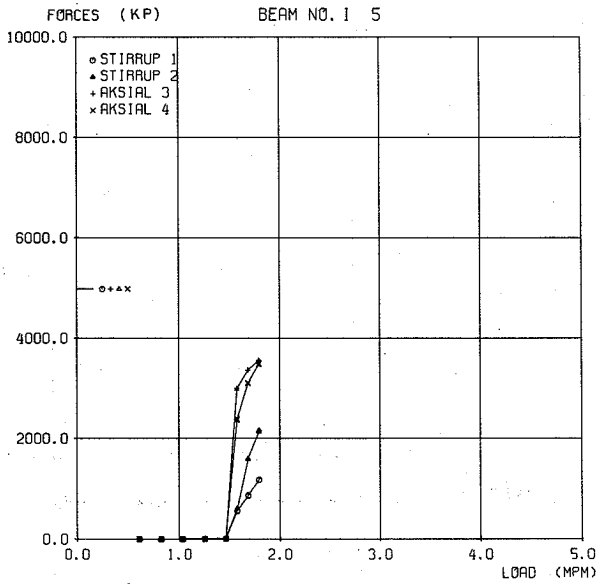
LOADING HISTORY
BEAM NO. III 1

ROTATIONS
BEAM NO. III 1

ROTATIONS

LOAD (MPM)   BEAM NO. III 1

AVERAGE GAUGE STRAINS

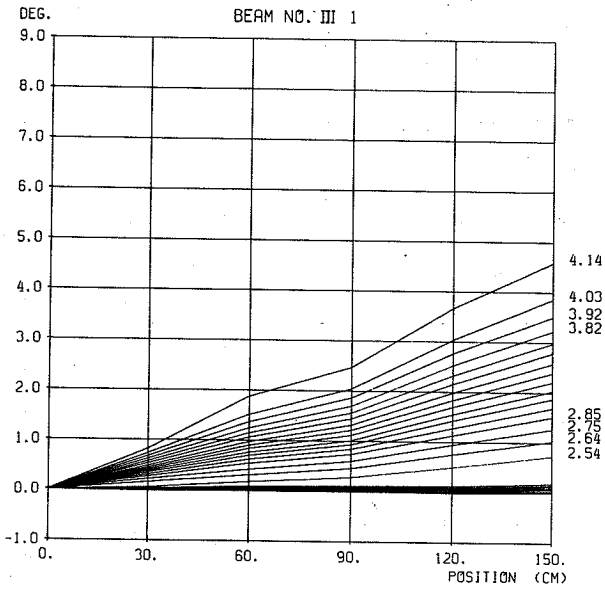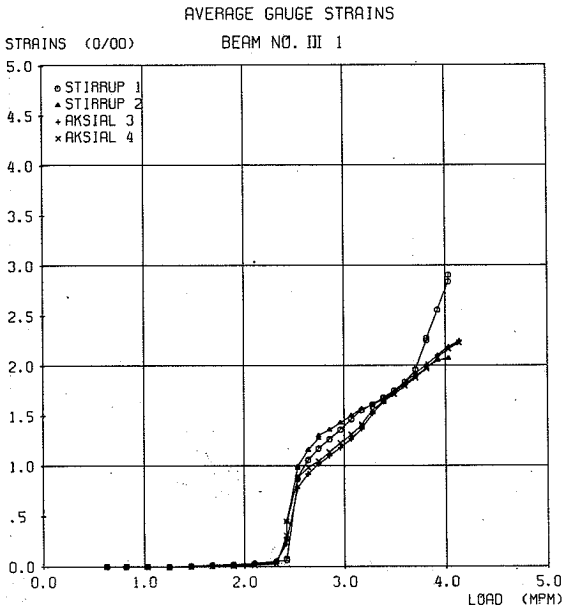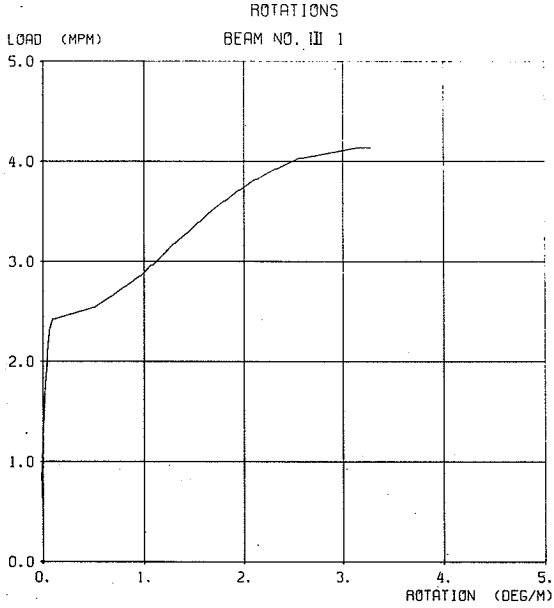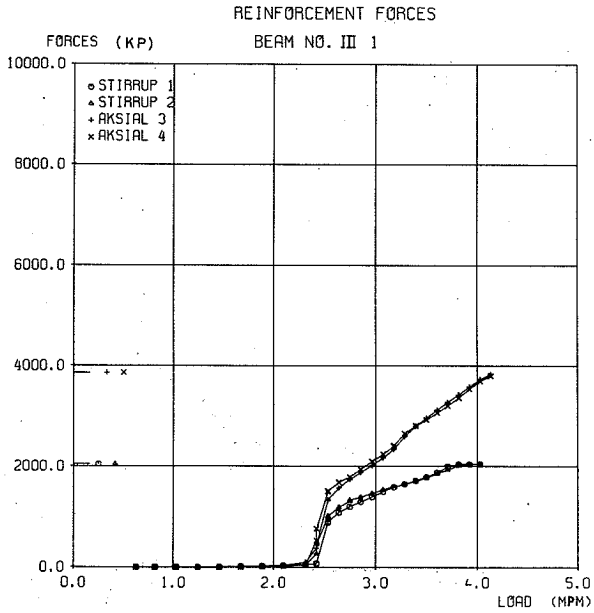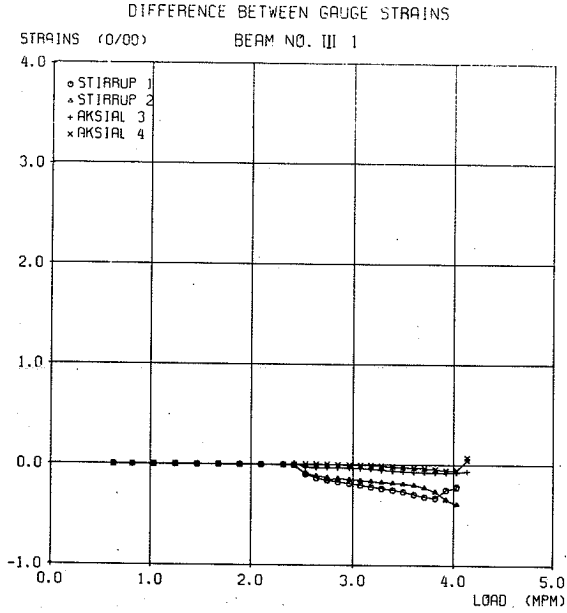STRAINS (0/00)   BEAM NO. III 1

DIFFERENCE BETWEEN GAUGE STRAINS
BEAM NO. III 1

REINFORCEMENT FORCES
BEAM NO. III 1

LOADING HISTORY
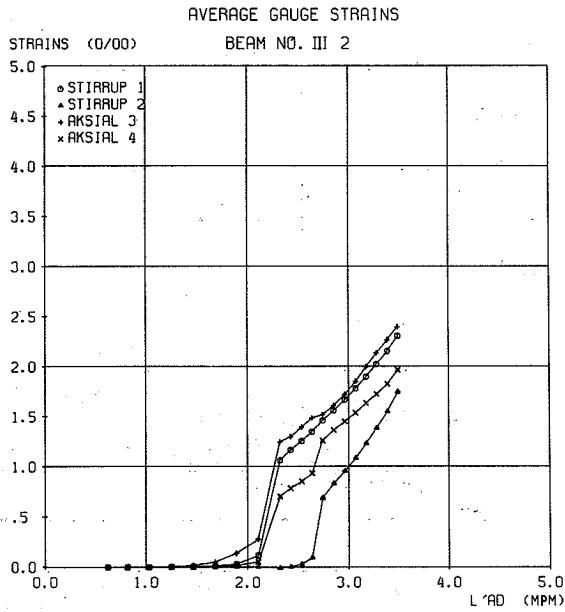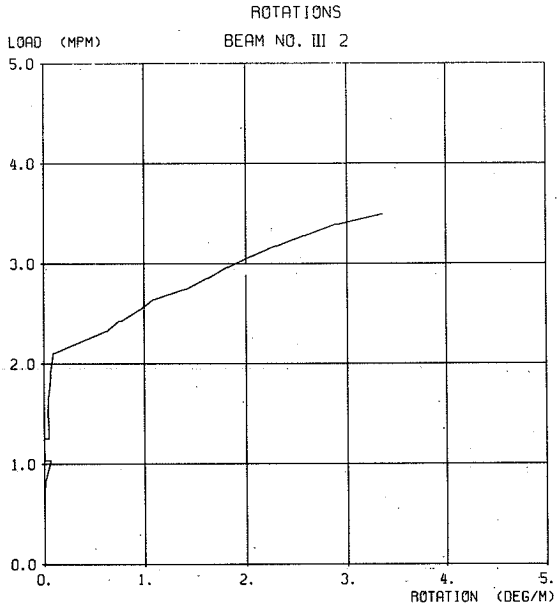
LOAD (MPM)   BEAM NO. III 2

5.0

⊙ SCANS

4.0

3.0

2.0

1.0

0.0
   0.   40.   80.   120.   160.   200.
                                          TIME (MIN)

ROTATIONS

DEG.   BEAM NO. III 2

9.0

8.0

7.0

6.0

5.0

4.0   3.50
                                                         3.39
                                                         3.28
3.0   3.18
                                                         3.07
                                                         2.85
2.0   2.75
                                                         2.54
1.0   2.43
                                                         2.32
0.0

-1.0
    0.   30.   60.   90.   120.   150.
                                          POSITION (CM)

ROTATIONS
BEAM NO. III 2

AVERAGE GAUGE STRAINS
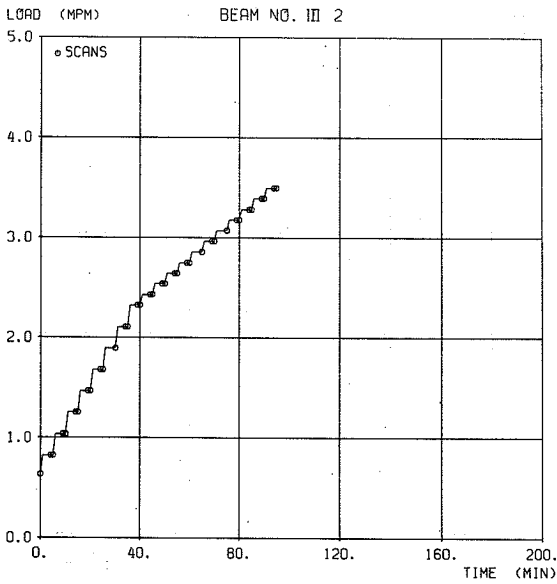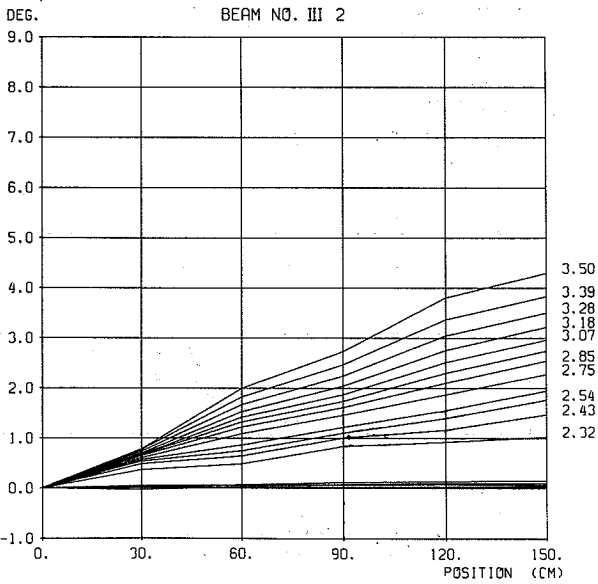BEAM NO. III 2

DIFFERENCE BETWEEN GAUGE STRAINS
BEAM NO. III 2

REINFORCEMENT FORCES
BEAM NO. III 2

LOADING HISTORY
BEAM NO. III 3

LOAD (MPM)

o SCANS

ROTATIONS
BEAM NO. III 3

DEG.

3.50
3.39
3.28
3.18
2.96
2.86
2.64
2.53
2.43

POSITION (CM)

ROTATIONS

LOAD (MPM)   BEAM NO. III 3

AVERAGE GAUGE STRAINS
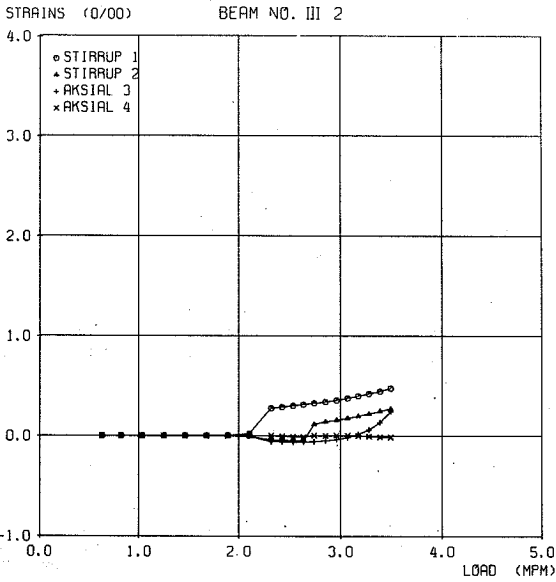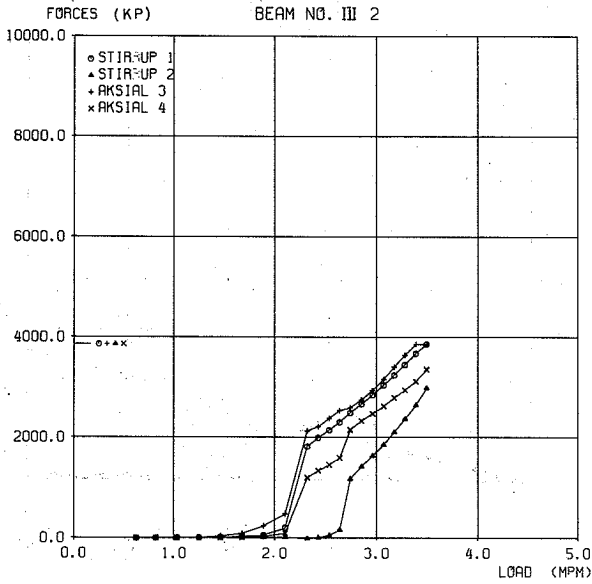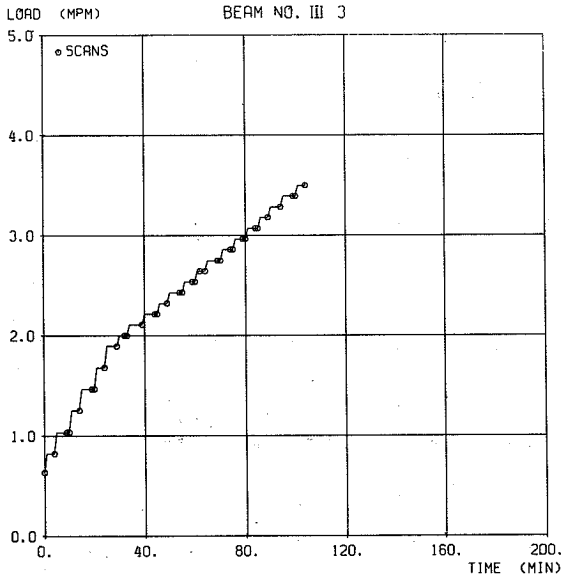
STRAINS (0/00)   BEAM NO. III 3

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (0/00)   BEAM NO. III 3

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. III 3

LOADING HISTORY
LOAD (MPM)   BEAM NO. III 4

⊙ SCANS

ROTATIONS
DEG.   BEAM NO. III 4

ROTATIONS

LOAD (MPM)   BEAM NO. III 4

AVERAGE GAUGE STRAINS
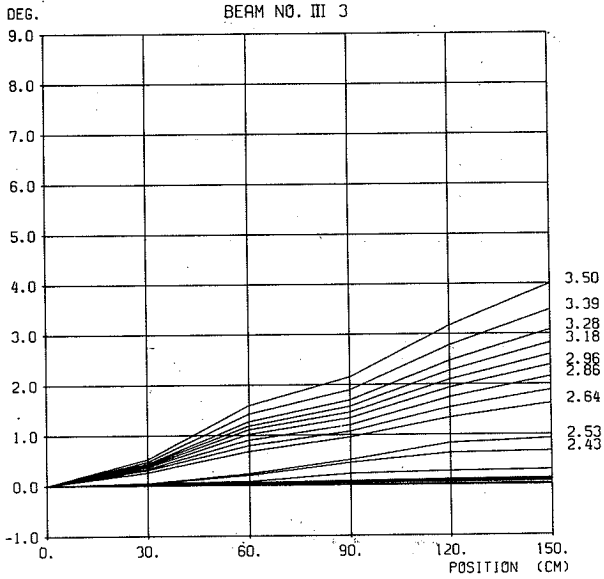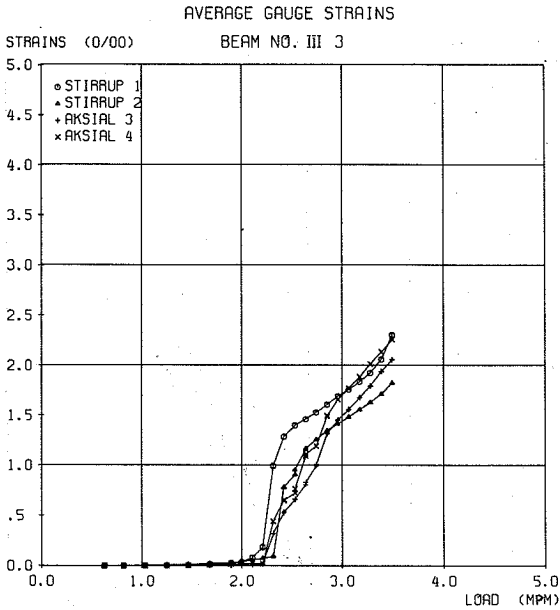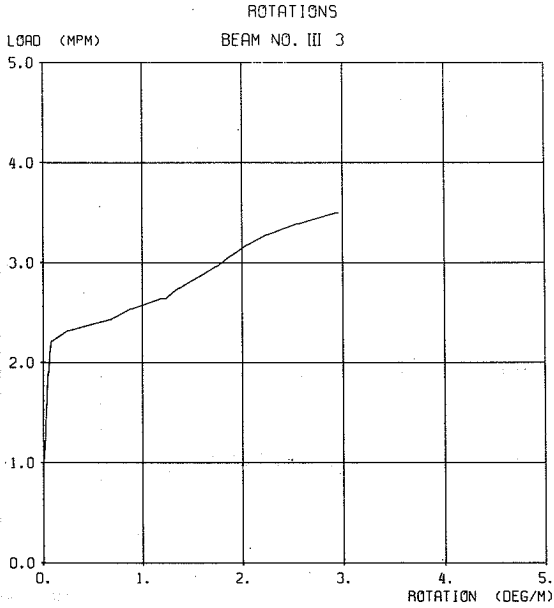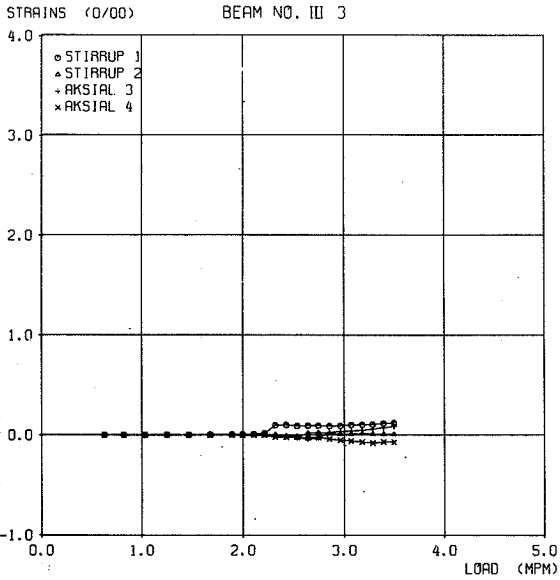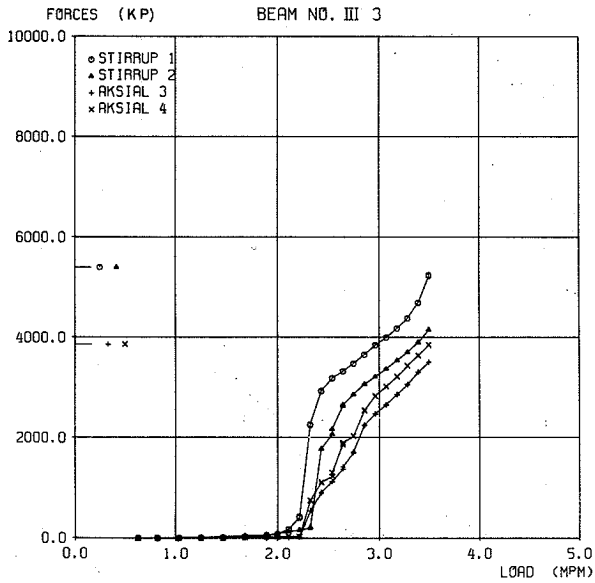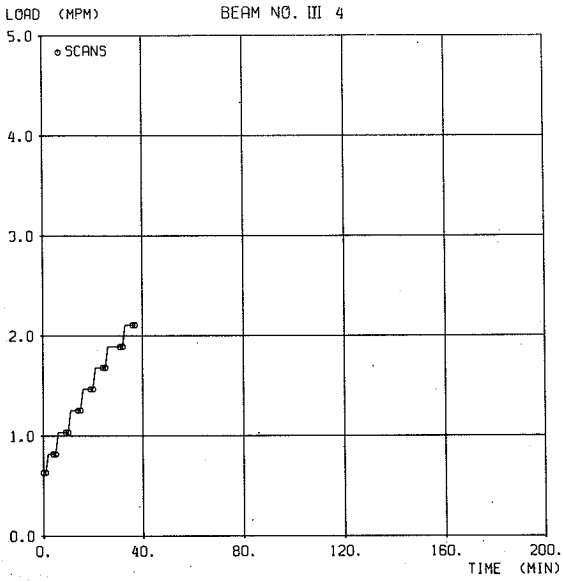
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DIFFERENCE BETWEEN GAUGE STRAINS
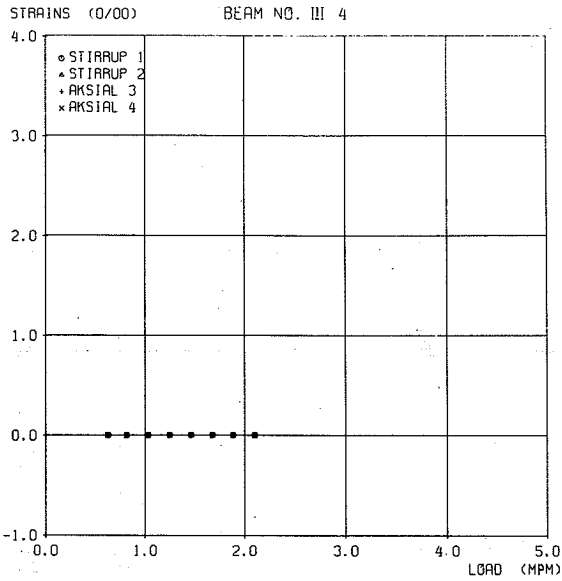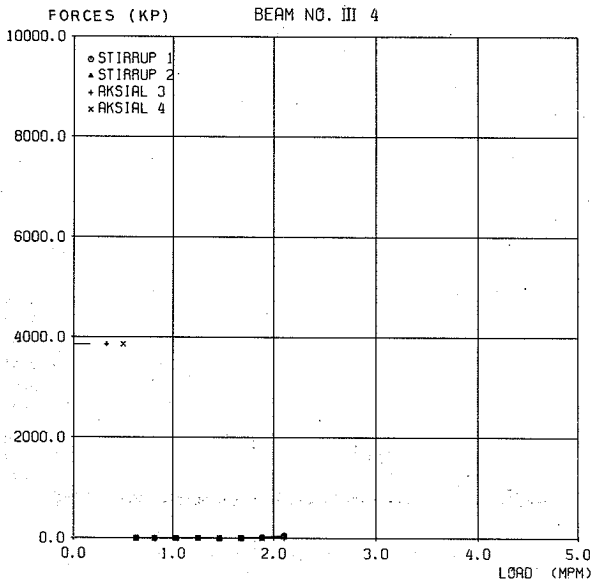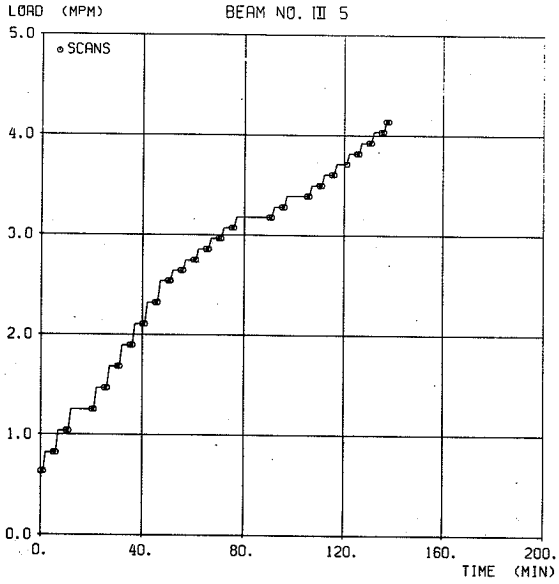
STRAINS (O/OO)   BEAM NO. III 4

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. III 4

LOADING HISTORY

LOAD (MPM)   BEAM NO. III 5

TIME (MIN)

ROTATIONS

DEG.   BEAM NO. III 5

POSITION (CM)

ROTATIONS

LOAD (MPM)   BEAM NO. III 5

ROTATION (DEG/M)

AVERAGE GAUGE STRAINS

STRAINS (0/00)   BEAM NO. III 5

LOAD (MPM)

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (O/OO)   BEAM NO. III 5

REINFORCEMENT FORCES
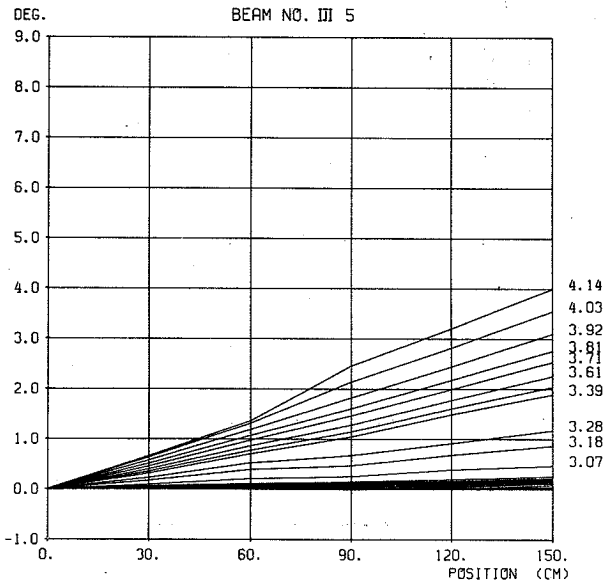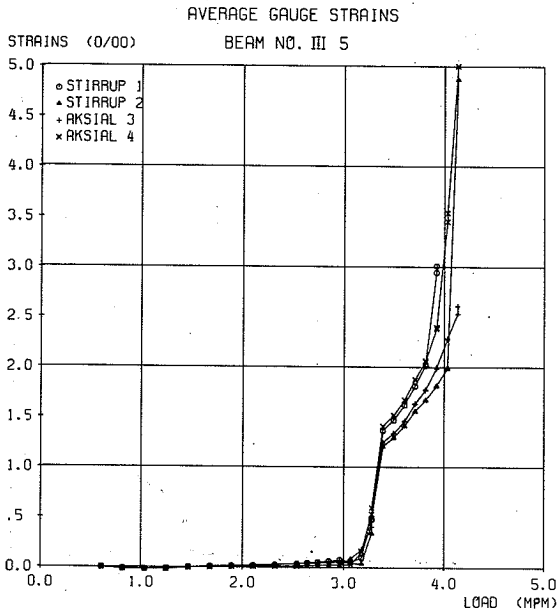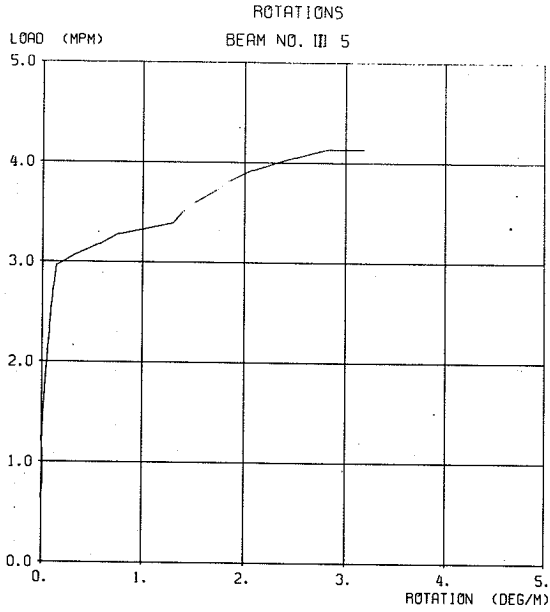
FORCES (KP)   BEAM NO. III 5
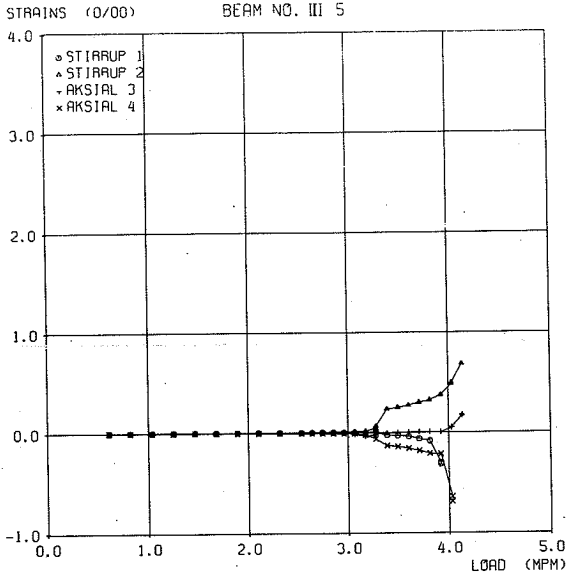
LOADING HISTORY
LOAD (MPM)   BEAM NO. III 6
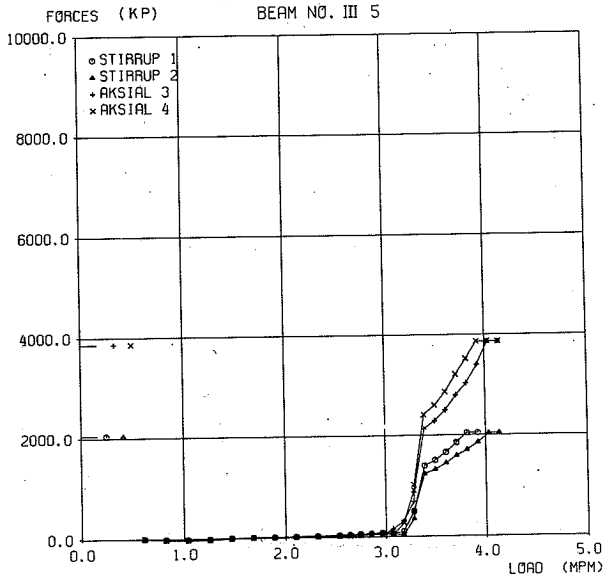
ROTATIONS
LOAD (MPM)   BEAM NO. III 6

ROTATIONS
BEAM NO. III 6

DEG.

9.0
8.0
7.0
6.0
5.0
4.0 ──────────────────────────── 3.60
                              3.49
3.0   3.39
                             3.28
                             3.18
2.0   3.07
                             2.96
1.0   2.86
0.0
-1.0
  0.   30.   60.   90.   120.   150.
                            POSITION (CM)

AVERAGE GAUGE STRAINS
BEAM NO. III 6

STRAINS (0/00)

5.0
     ⊙ STIRRUP 1
     ▲ STIRRUP 2
4.5 + AKSIAL 3
     × AKSIAL 4
4.0
3.5
3.0
2.5
2.0
1.5
1.0
 .5
0.0
  0.0   1.0   2.0   3.0   4.0   5.0
                              LOAD (MPM)

DIFFERENCE BETWEEN GAUGE STRAINS

STRAINS (O/OO)   BEAM NO. III 6

Legend:
- o STIRRUP 1
- ▲ STIRRUP 2
- + AKSIAL 3
- × AKSIAL 4

LOAD (MPM)

REINFORCEMENT FORCES

FORCES (KP)   BEAM NO. III 6

Legend:
- o STIRRUP 1
- ▲ STIRRUP 2
- + AKSIAL 3
- × AKSIAL 4

LOAD (MPM)

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