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Dome Effect in Reinforced Concrete Slabs

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CONTENTS

RESUMÉ

Det er almindeligt kendt, at bæreevnen af armerede betonplader med vandret fastholdelse ved randene kan være flere gange større end den bæreevne, der fås fra brudlinieteorien.

Det illustreres, hvordan en teori for armerede betonplader med vandret fastholdelse ved randene kan opstilles, når det antages, at materialerne kan betragtes som værende stift plastiske. De teoretiske udtryk for bærevenen gælder for rektangulære plader med vilkårligt forhold mellem sidelængderne. Udtrykkene kan derfor i grænserne blive brugt til både kvadratiske og enkeltspændte plader.

De teoretiske bæreevneudtryk sammenlignes med forsøgsresultater, og overensstemmelsen mellem teori og forsøg findes at være tilfredstillende. Til praktisk brug opstilles på baggrund af forsøgsbehandlingen simple udtryk til bestemmelse af bæreevnen.

SYNOPSIS

It is well known that the load carrying capacity of reinforced concrete slabs with horizontal restraints can be several times the load carrying capacity found by the yield line theory.

Demonstrated here is how a theory, based on the theory of perfectly plastic materials, can be developed for reinforced concrete slabs with horizontal restraints at the edges. The theoretical expressions for the load carrying capacity are valid for rectangular slabs with an arbitrary ratio between the two sides. Therefore, the expressions in the limits can be used for both square slabs and one-way slab strips.

The theory is compared with test results and the agreement is found to be satisfactory. In the light of the test treatment simple formulas for practical use for the load carrying capacity are set up.

1. NOTATION

a	The smaller side length of a rectangular slab.
b	The larger side length of a rectangular slab.
d	Effective depth for the bottom side reinforcement.
d'	Effective depth for the top side reinforcement.
f_c	Uniaxial compressive strength of concrete (measured on 150 x 300 mm cylinders).
f_t	Uniaxial tensile strength of concrete.
f_y	Yield strength of reinforcement.
h	Depth of slab.
k	Load and geometrical factor.
m	Dimensionless moment
m_J	Moment, $m_J = h^2 f_c \cdot m$.
m_m	Dimensionless membrane moment.
m_M	Membrane moment, $m_M = h^2 f_c \cdot m_m$
m_{JM}	Moment, the sum of m_J and m_M .
m_n	Dimensionless moment of reduction.
p	Uniform load.
$\dot{u}$	Increment of the deflection.
v	Variable length.
w	Deflection in slab middle.
x	Variable length
y	Variable length
z	Variable length
A	Area.
A_c	Area of concrete.
A_s	Cross-sectional area of the bottom side reinforcement.
A_s'	Cross-sectional area of the top side reinforcement.
A_1	Area in the "area rule" (upper area).
A_2	Area in the "area rule" (lower area).
H	Horizontal force.
I	Instantaneous center of rotation.
P, P_T	Total load on slab.
P_{test}	Total load obtained in test.
P_{theory}	Total theoretical load carrying capacity.
P_J	Total load carrying capacity according to the yield line theory.

P_M	Total membrane load carrying capacity.
P_N	Total reduction in the load carrying capacity.
W_e	External work.
W_i	Internal work.
γ	Relative effective depth for the bottom side reinforcement, $\gamma = d/h$.
γ'	Relative effective depth for the top side reinforcement, $\gamma' = d'/h$.
δ	Relative deflection in slab middle, $\delta = w/h$.
$\bar{\delta}$	Average value for the relative deflection.
δ'	$\delta' = \delta/\kappa$.
δ'_z	$\delta'_z = \delta/\kappa_z$.
δ'_v	$\delta'_v = \delta/\kappa_v$.
κ	$\kappa = (\frac{1}{2}\frac{a}{x} + 2\frac{x}{a}) / (\frac{1}{2}\frac{a}{x} + \frac{b}{a})$.
κ_v	$\kappa_v = (\frac{a}{v} + \frac{v}{a-z}) / (\frac{a}{v} + \frac{1}{2}\frac{b}{a-z})$.
κ_z	$\kappa_z = (\frac{1}{2}\frac{a}{x} + 2\frac{x}{a} + 2\frac{y}{a}) / (2\frac{b}{a} + \frac{1}{2}\frac{a}{x})$.
λ	Slenderness of slab $\lambda = (a+b)/(2\,h)$.
ν	Effectiveness factor .
ξ	Compression zone relative to the depth h .
σ_y	Tensile and compression yield strength of an arbitrary material.
Δ	Horizontal displacement relative to the depth h .
Φ	Mechanical degree of bottom side reinforcement $\Phi = \frac{A_s f_y}{h f_c}$.
Φ'	Mechanical degree of top side reinforcement $\Phi' = \frac{A_s f_y}{h f_c}$.

2. INTRODUCTION

The yield line theory has been used for many years to calculate the ultimate strength of reinforced concrete slabs. This theory was developed primarily by Ingerslev [1], [2] and Johansen [3], [4]. Johansen [3, 4] was aware of the tensile membrane action, but the first in the Western World to demonstrate the great effect of restrained edges was Ockleston [5, 6]. During a serie of tests on a condemned building, Ockleston became aware of a breakdown of the rigid plastic first-order theory for internal slab parts. However, the membrane effect was already taken into account in the 1939 Russian code of practice for reinforced concrete structures. Gvozdev explained in [7] why the Russian code for reinforced concrete permitted a 20% increase in the calculated ultimate load for interior slabs.

In normal calculations based on the theory of plasticity, the changes in geometry are neglected (first-order theory). As known, this normally gives good results when comparing theory with tests. Since slabs are often rather flexible structures, the changes in geometry sometimes have a considerable effect on the load carrying capacity. These effects are called membrane effects. One often speaks about a compressive membrane effect which predominates at small deflections and of tensile membrane effect, which dominates at larger deflections. In a simply supported slab only tensile membrane action exists, but in a slab with horizontal restraints along the edges, both compression and tensile membrane action may be found, depending on the size of the deflection.

A simple explanation of the compression membrane action is in order. In pure bending of reinforced concrete with small steel ratios, the neutral axis at failure is very close to the surface with compression in the concrete. This means that pure bending is accompanied by extensions of the middle surface. If the support conditions are such that these deformations cannot take place (horizontal restraints), failure corresponding to pure bending cannot occur. The neutral axis must be brought close to mid-depth of the section, which requires that large compressive membrane forces must be supplied by the support. The compressive zones in this case therefore are in the order of half the slab thickness, and the ultimate moment and therefore the total load, are very high.

The effectiveness or the degree of horizontal restraints is an important parameter which has to be taken into account. Tests on rectangular slabs, when horizontal displacements are prevented along the edges, can be divided into two groups; the first group having "normal" restraints and the second group having "rigid" horizontal restraints. A normal restraint is a restraint which does not prevent horizontal displacements, but can still offer resistance to the displacement. This kind of restraint is comparable to slabs as they are built in normal practice. A rigid restraint is a restraint which prevents any horizontal displacement at the edge.

A rectangular slab having normal horizontal restraints at all four edges typically has an ultimate strength 2 to 3 times the normal yield line strength. For a slab with rigid restraints, the ultimate load is typically 3 to 5 times the normal yield line strength. If the ultimate strength due to membrane action could be taken into account in a simple way it would be possible to utilize these large reserves.

Many attempts to create expressions for the load carrying capacity for a slab with horizontal restraints have been carried out. Wood [8], Christiansen [9], Park [10], Sawczuk [11, 12], Janas & Sawszuk]13], Janas [14], Morley [15], Calladine [16], Massonet [17], Birke [18], Bræstrup [19], Bræstrup & Morley [20], Christiansen [21, 22] and Eyre & Kemp [23] name just a few. Beyond this, many tests have been carried out both on full-scale slabs and on model slabs. The theoretical expressions for the load carrying capacity derived from these attempts are in general very compliceted and are only valid for special cases. Therefore, they cannot be used in practical calculations. For more detailed historical reviews the reader is referred to Bäcklund [24] and Bræstrup [19].

As there exist large reserves in the load carrying capacity due to membrane action, and because a simple method based on a consistent theory for the total strength of a slab with horizontal restraints does not exist, it will therefore in this paper be attempted to find relatively simple expressions for the total load carrying capacity in a slab with horizontal restraints along the edges.

In [16], Calladine set up a three-dimensional theory for reinforced concrete slabs. In the case of membrane action in slabs, Calladine's method is very convincing. Many problems can be managed in a simple way. In Calladine's method the calculations are carried out according to the flow theory (dealing with increments for the deformations as in normal upper bound calculations).

Since Calladine only gave expressions for circular slabs, his formulas cannot be used in practical calculations. Here Calladine's method is extended to rectangular slabs with horizontal restraints along the edges.

The work described in the following was carried out as a Master Thesis by Andreasen [25]. For a more detailed description the reader is referred to this report.

3. BASIC ASSUMPTIONS

The theory of plasticity is applied. In the calculations, the upper bound theorem will be used to develop expressions for the ultimate load carrying capacity. In this theorem the external work done by the load in a failure mechanism is equated to the internal work dissipated in the structure. The yield line concept for plane stress is used to find the dissipation.

According to the upper bound theorem, the load found by the work equation for a geometrically possible failure mechanism is greater than or equal to the yield load.

The concrete is assumed to be a rigid plastic material. Coulomb's failure criterion together with limitation of the tensile strength, the modified Coulomb failure criterion, is used as yield condition. The tensile strength is considered to be zero.

The well known fact that concrete is not perfectly plastic is taken into account by reducing the uniaxial compression strength f_c by the effectiveness factor ν . The effective plastic strength is then taken to be $f_{cp} = \nu f_c$.

The reinforcement is assumed to be rigid-plastic and to carry only longitudinal stresses. Thus the dowel effect is neglected.

A more thorough summary of the assumptions can be found in M.P. Nielsen [26]. This book also presents various solutions for the load carrying capacity of different concrete structures.

4. THEORETICAL LOAD CARRYING CAPACITY

In conventional slab calculations for determining the ultimate strength, the structure is regarded as a two-dimensional structure. In the case of slabs with membrane action this method leads to complicated calculations. If instead the slab is considered to be a three-dimensional structure, the calculations can be carried out in a very simple way. This was shown by Calladine [16]. The three-dimensional method will be denoted "Calladine's method" in this paper. The method is described below and the concept of "membrane moment" is defined. Calladine's method is then used to find an expression for the load carrying capacity of a rectangular slab with horizontal restraints at the edges. Rectangular slabs with horizontal restraints on three sides only are then briefly treated. Finally, a short discussion and comparison with other expressions for the load carrying capacity is carried out.

4.1 CALLADINES METHOD

The basic principle in Calladine's method will be mentioned briefly. For a more detailed description the reader is referred to Calladine [16].

The slab is regarded as a three-dimensional structure. The material is assumed to be rigid, perfectly plastic, with the yield strength σ_y . The upper bound theorem is used.

Considering a circular slab with radius a , thickness h , simply supported at its edge and carrying a central load P (see figur 1), the load carrying capacity can be written

$$P = 2\pi\sigma_y \frac{1}{a} \cdot \int_A |y| \, dA = 2\pi\sigma_y \frac{1}{a} \circ S \qquad (1)$$

Symbols can be found in figure 1.

Normal yield line theory indicates a conical failure mechanism, which is why this mechanism is also used here.

It is of course assumed that punching failure does not occur.

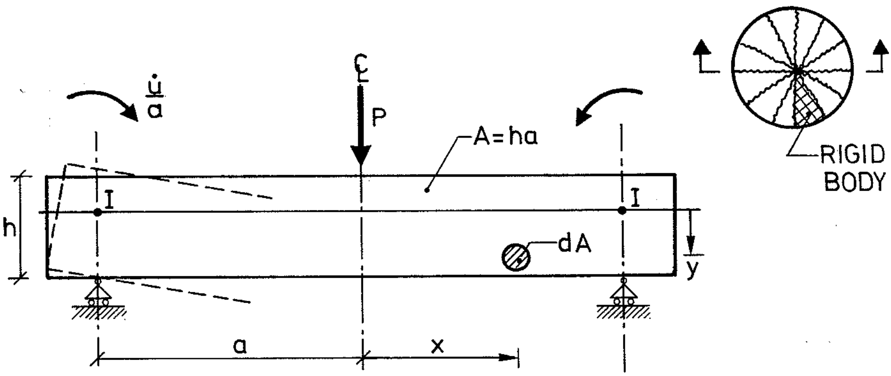

Figure 1: Simply supported circular slab with a central load.

The displacements are assumed to take place as a rotation $\dot{\omega}$ (increment), about a point, I , which lies on a vertical line through the support point. The best position of I is the position which will minimize the value of P in (1).

From (1), the best upper bound solution (lowest value of P) occurs when the value S of the integral is at a minimum. It can be shown that for arbitrary areas, the integral is at a minimum when the line I-I divides the cross-section into two equal areas. Thus, the best solution occurs when the area over the rotation line, A_1, is equal to the area under the rotation line, A_2. The best position of I can then be found by equating the two areas A_1 and A_2. This will be denoted as the "area rule". If the yield stress throughout the section is not the same, the different areas must be multiplied by the actual yield stress. If the rigid bodies in the failure mechanism do not have the same rotation, the areas must be multiplied by the corresponding rotation.

If the slab has horizontal restraints along the edge, an additional term to the right hand side of (1) appears. This term takes into account the work done at the edge. It can be written as

$$2\,\pi\,\sigma_y \frac{1}{a} \int_h a|y|\,dy \qquad (2)$$

The minimum for the load carrying capacity can still be found by the area rule. The area over the rotation line should be equal to the area under the line.

In the following section Calladine's method will be illustrated in a simple case.

4.2 MEMBRANE MOMENT

An unreinforced slab strip with a line load and horizontal restraints at the edges is considered, see figure 2.

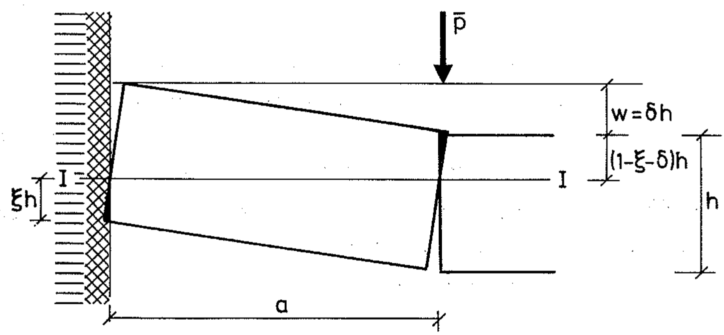

Figure 2: Slab strip with a deformation w *(absolute value) under the line load.*

Using Calladine's method on the deformed slab strip, a drawing similar to the one shown in figure 3 is useful. In this figure the section at the edge and at the line load is shown.

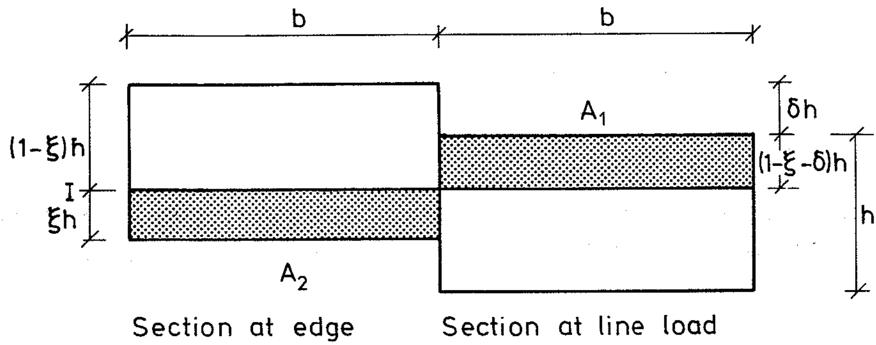

Figure 3: Section at the edge and at the line load for a deformed slab strip (see also figure 2).

The toned areas in figure 3 indicate compression areas in the concrete.

Giving the deformed slab an incremental deflection , $\dot{u}$, the areas A_1 and A_2 can be written

$$A_1 = \nu f_c (1 - \xi - \delta) hb \frac{\dot{u}}{a} \quad , \quad A_2 = \nu f_c \cdot \xi hb \frac{\dot{u}}{a} \tag{3}$$

Using the area rule $A_1 = A_2$, the best value for ξ can be found to be

$$\xi = \frac{1}{2}(1 - \delta) \tag{4}$$

The integral in (1) and (2) given by S can, in this case, be written

$$S = \frac{1}{2}\xi^2 h^2 b + \frac{1}{2}(1 - \xi - \delta)^2 h^2 b \tag{5}$$

The internal work is found to be

$$W_i = \frac{\dot{u}}{a} \cdot \nu f_c \circ S \tag{6}$$

and the external work is

$$W_e = \bar{p}\, b\, \dot{u} \tag{7}$$

Inserting (4) and (5) in (6) and using the upper bound theorem on (6) and (7), the expression for the load carrying capacity is found to be

$$P = \bar{p}\, b = \frac{b}{a} \cdot \frac{1}{4} \nu f_c h^2 (1 - \delta)^2 \tag{8}$$

Introducing the membrane moment, m_M , as

$$m_M = \frac{1}{4} \nu f_c h^2 (1 - \delta)^2 \tag{9}$$

the expression for the total load carrying capacity (8) can be written

$$P = \frac{b}{a} \cdot m_M \tag{10}$$

A lower bound calculation gives the same result, see for example Nielsen [26], who has set up the formula for $\delta = 0$.

The expression for the membrane moment (9) can be rewritten

$$m_M = (\tfrac{1}{2} h (1 - \delta)) \nu f_c \cdot (\tfrac{1}{2} h (1 - \delta)) \tag{11}$$

It can be seen that the membrane moment corresponds to a moment from stresses in two cross sections. The compression zones have a depth $\frac{1}{2} h(1-\delta)$ and the distance between the resultant forces is $\frac{1}{2} h(1-\delta)$. It should be noted that stresses from two different cross sections give the "moment".

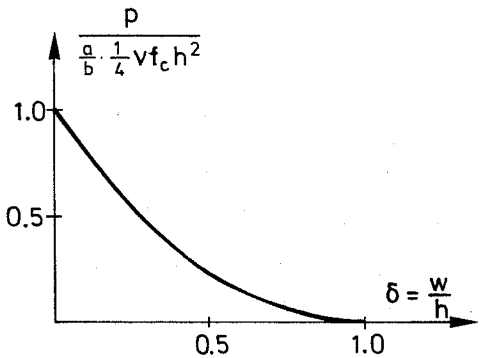

Figure 4: The load carrying capacity for an unreinforced concrete slab strip with horizontal restraints at the edges as a function of the relative displacement under the line load.

In figure 4 the load carrying capacity (10) is given as a function of the relative displacement δ. The load carrying capacity approaches zero when the relative displacement, δ, approaches 1. An explanation for this can be given by looking at the expression for m_M (11) and figure 2. The effective depth of the cross section approaches zero, whereby a compression arch cannot be established.

4.3 RECTANGULAR SLABS

In this section expressions for the load carrying capacity for rectangular slabs with horizontal restraints at all edges will be set up. Expressions for slabs reinforced isotropically and without compression reinforcement will be developed. Orthotropically reinforced slabs and/or slabs with compression reinforcement can be found by the presented method, but the calculations and the expressions are very complicated. It will also be shown that slabs with arbitrary reinforcement can be treated by "simple expressions" for the load carrying capacity. The slab and the yield line pattern used are shown in figure 5.

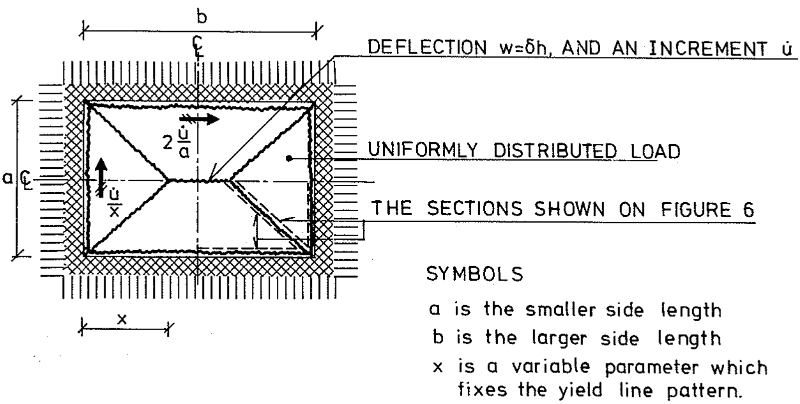

Figure 5: Rectangular slab with horizontal restraints on all edges, and uniform load.

The load is assumed to be uniform, but as it will appear, most of the calculations and expressions are independent of the type of load if the applied load will cause the same yield line pattern.

The cross sections in a part of the yield lines are shown in figure 6. The yield lines represented on figure 6 are shown with a dotted line on figure 5.

The internal and external work can then be written as

$$W_i = 4S \cdot f_c \qquad W_e = \frac{1}{6}\, pab\, (3 - 2\,\frac{x}{b})\,\dot{u} \tag{12}$$

where S is the work from the cross section shown in figure 6, (quarter of the whole slab). Here ν and $\dot{u}$ are included in S (see (17) and (21)).

Using the upper bound theorem on (12), the total dimensionless load carrying capacity can be written

$$\frac{P}{h^2 f_c} = \frac{pab}{h^2 f_c} = \frac{24\ S}{\dot{u}h^2\,(3 - 2\,\frac{x}{b})} \tag{13}$$

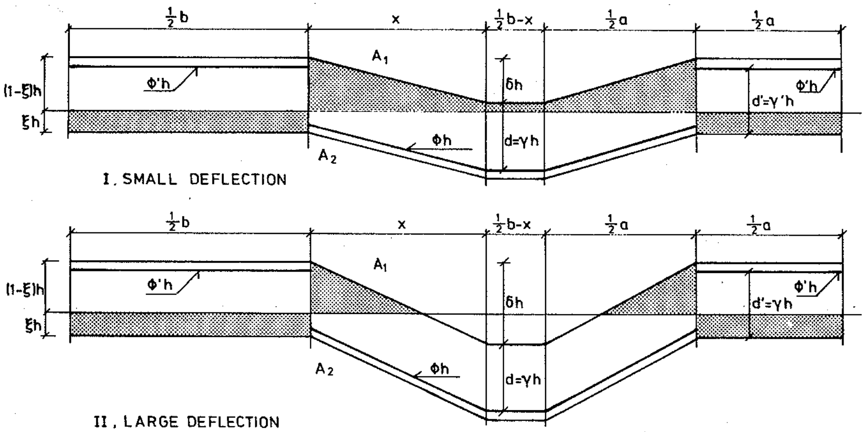

Figure 6: Deformed rectangular slab. Cross sections in yield lines for a small and a large deformation, respectively.

The expression for S in a slab having a small deflection is considered first, see figure 6.I. The areas A_1 and A_2 in the area rule can in this case be written

$$A_1 = (1-\xi-\delta)h\nu(\frac{1}{2}a\frac{\dot{u}}{x}+\frac{1}{2}b\frac{2\dot{u}}{a})+\frac{1}{2}\delta h\nu(x\frac{2\dot{u}}{a}+\frac{1}{2}a\frac{\dot{u}}{x})+\Phi'h(\frac{1}{2}a\frac{\dot{u}}{x}+\frac{1}{2}b\frac{2\dot{u}}{a})$$

$$A_2 = \xi h\nu(\frac{1}{2}a\frac{\dot{u}}{x}+\frac{1}{2}b\frac{2\dot{u}}{a})+\Phi h(\frac{1}{2}a\frac{\dot{u}}{x}+\frac{1}{2}b\frac{2\dot{u}}{a}) \qquad (14)$$

where $\dot{u}$ is an incremental deflection in the center of the slab. Using the area rule, ξ can be found to be

$$\xi = \frac{1}{2}(1-\frac{1}{\nu}(\Phi-\Phi')-\frac{1}{2}\delta(2-\kappa)) \qquad (15)$$

where

$$\kappa = \frac{\frac{1}{2}\frac{a}{x}+2\frac{x}{a}}{\frac{1}{2}\frac{a}{x}+\frac{b}{a}} \qquad (16)$$

The parameter κ in (16) is a measure of the extent to which the slab deformation is governed by the triangles compared to that of a strip.

The parameter S is given by

$$S = \frac{1}{2} h^2 (\frac{1}{2}\frac{a}{x} + \frac{b}{a}) \dot{u} [\nu\xi^2 + \nu(1-\xi-\delta)^2 + \nu\delta(1-\xi-\frac{2}{3}\delta)\kappa$$

$$+ 2\Phi(\gamma - (1-\xi)) + 2\Phi'(\gamma'-\xi) + \Phi\delta(2-\kappa)] \tag{17}$$

(15) - (17) is only valid when $(1-\xi-\delta) \geq 0$. By inserting (15) into this condition we find

$$\delta \leq \frac{2}{2+\kappa} (1 + \frac{1}{\nu}(\Phi-\Phi')) \tag{18}$$

If this condition is not fulfilled, figure 6.II must be used. ξ in this case can be determined by

$$\xi = 1 + \delta' - \sqrt{\delta'(\delta' + 2 + \frac{2}{\nu}(\Phi - \Phi'))} \tag{19}$$

where

$$\delta' = \delta/\kappa \tag{20}$$

and the parameter S is given by

$$S = \frac{1}{6} h^2 (\frac{1}{2}\frac{a}{x} + \frac{b}{a}) \dot{u} [\nu\kappa \frac{(1-\xi)^3}{\delta} + 3\nu\xi^2 + 3\Phi\delta(2-\kappa)$$

$$+ 6\Phi(\gamma - (1-\xi)) + 6\Phi'(\gamma'-\xi)] \tag{21}$$

Inserting (15) into (17) and (19) into (21) and inserting these expressions for S into (13) the total load carrying capacity can be written as

$$\frac{P}{h^2 f_c} = 24 \frac{\frac{1}{2}\frac{a}{x} + \frac{b}{a}}{\frac{a}{b}(3\frac{b}{a} - 2\frac{x}{a})} [m + m_n + m_m] \tag{22}$$

where, for $\delta \leq \frac{2}{2+\kappa} (1 + \frac{1}{\nu}(\Phi - \Phi'))$,

$$m = \Phi(\gamma - \frac{\Phi}{2\nu}) + \Phi'(\gamma' - \frac{\Phi'}{2\nu}) \qquad (23)$$

$$m_n = \frac{1}{2\nu}\Phi\Phi' + \frac{1}{4}\delta(2-\kappa)(\Phi+\Phi') - \frac{1}{2}\Phi(1 - \frac{\Phi}{2\nu}) - \frac{1}{2}\Phi'(1 - \frac{\Phi'}{2\nu}) \qquad (24)$$

$$m_m = \frac{\nu}{4}(1 - (2-\kappa)\delta + \frac{1}{12}(12 - 4\kappa - 3\kappa^2)\delta^2) \qquad (25)$$

and for $\delta > \frac{2}{2+\kappa}(1 + \frac{1}{\nu}(\Phi - \Phi'))$,

$$m = \Phi(\gamma - \frac{\Phi}{2\nu}) + \Phi'(\gamma' - \frac{\Phi'}{2\nu}) \qquad (26)$$

$$m_n = \frac{\nu}{3}(\sqrt{\delta'(\delta'+2)^3} - \sqrt{\delta'(\delta'+2+\frac{2}{\nu}(\Phi-\Phi'))^3}) + \frac{1}{2\nu}(\Phi^2+\Phi'^2) + \frac{1}{2}(\Phi(2+(2-\kappa)\kappa)\delta' - 2\Phi'(1+\delta')) \qquad (27)$$

$$m_m = \frac{\nu}{6}(2\delta'^2 + 6\delta' + 3 - 2\sqrt{\delta'(\delta'+2)^3}) \qquad (28)$$

As it can be seen from (23) and (26), m is the normal moment and as in the previous section, m_m stands for membrane moment. m_n can be looked at as a reduction, caused by the combined bending and membrane action.

Inserting $a = b$ and $x = \frac{1}{2}a$ in (13) and (22) - (28) the expressions correspond to those valid for the square slab. If b/a is infinite, κ will approach zero and the factor in front of the bracket in (22) approaches $8 \cdot b/a$. In this case the slab can be considered as a one way slab (slab strip) and the expression for the membrane moment, (25), corresponds to that found in the previous section, equation (9).

The load carrying capacity is a function of the variable parameter x . If m , m_n and m_m were independent of x a normal minimizing would be possible, as the factor in front of the bracket in (22) is the same as found by a normal yield line calculation. Since m_n and m_m are not independent of x the calculations will be incorrect if x is found by the use of the normal expressions. A theoretically correct minimizing of (22) with respect to x is possible, but the expression obtained for x is complicated. Therefore the load obtained with

$$\frac{x}{a} = \frac{1}{2}\,[\sqrt{3 + (\frac{a}{b})^2} - \frac{a}{b}] \tag{29}$$

which corresponds to the normal yield line calculation has been compared with the optimal solution. It was found that the load obtained with (29) was between 0.5% to 5% larger than the optimal load. A difference of about 5% was obtained when $\Phi = \Phi' = 0$. For slabs with normal reinforcement degree, the difference was about 1% and thus of little importance.

Due to the satisfactory results obtained with the normal value for x , it was assessed that use of this value would give results which could be useful for practical calculations. Therefore equation (29) is used below to determine x for rectangular slabs with horizontal restraints at the edges. The expression for the total load carrying capacity (22) can then be written

$$\frac{P}{h^2 f_c} = \frac{24}{\frac{a}{b}\left(\sqrt{3+(\frac{a}{b})^2} - \frac{a}{b}\right)^2}[m + m_n + m_m] \tag{30}$$

where m, m_n and m_m are given by (23) - (28).

The load carrying capacity, (30), is shown as a function of the relative deflection, δ , for some cases in figure 7.

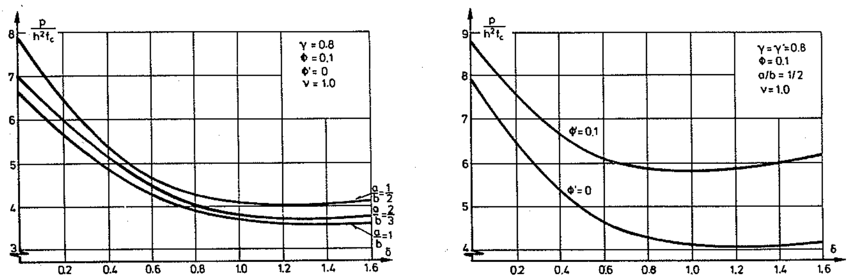

Figure 7: The load carrying capacity as a function of the relative deflection, δ , for rectangular slabs with horizontal restraints at the edges.

As it can be seen from figure 7, the total load carrying capacity is increased when a/b is decreasing, which could be expected. Besides the load carrying capacity is increased when Φ' (or Φ) is increased, see figure 7.II. The great similarity between the different curves must be noticed. Moreover the flat part of the curves for δ between 0.7 and 1.6 should be noted. In figure 8

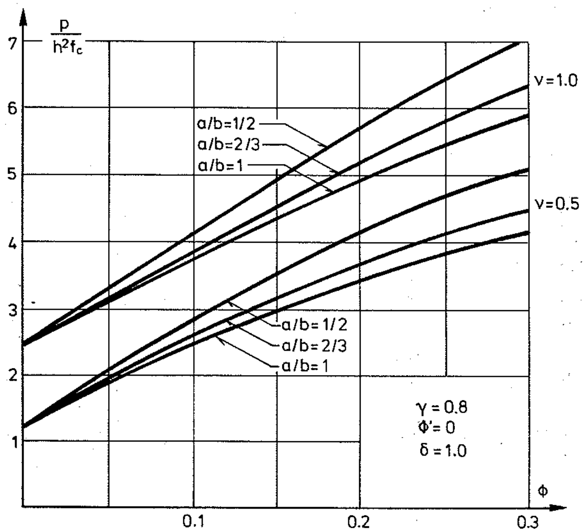

Figure 8: The total load carrying capacity as a function of the mechanical reinforcement degree Φ for rectangular slabs.

the total load carrying capacity is shown as a function of the mechanical reinforcement degree, Φ . It is again seen that the total load carrying capacity is increased for increasing Φ or decreasing a/b . Moreover it is seen that the load is increased for increasing ν . The small difference between the curves for small Φ should also be noted.

4.4 RECTANGULAR SLABS WITH HORIZONTAL RESTRAINTS ON THREE SIDES ONLY

Calladine's method can be applied to rectangular slabs with normal supports along the edges and horizontal restraints on three sides. The calculations and expressions are slightly more complicated than for the slabs with horizontal restraints on four sides, but the principles are the same. One must distinguish between slabs with a short and a long side without horizontal restraint. The considered slabs are shown on figure 9.

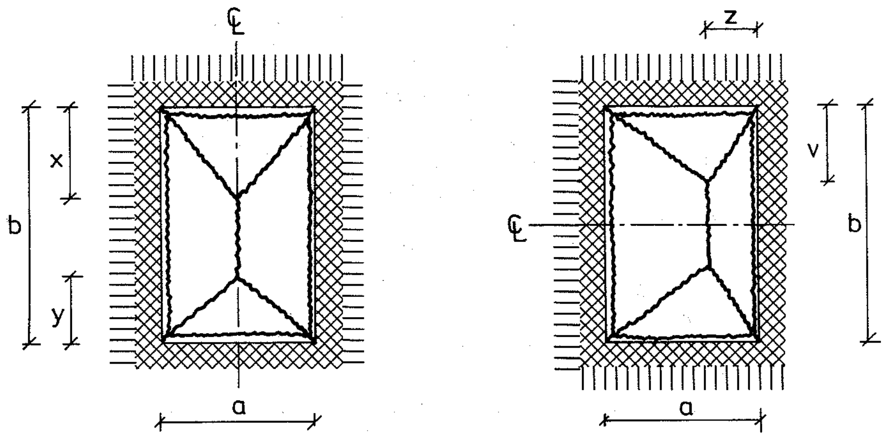

Figure 9: Rectangular slabs with horizontal restraints on three sides and the corresponding yield line patterns.

The load is assumed to be uniform, just as in the previous section. The yield line patterns shown on the figure are most likely to occur when the ratio between the reinforcements in the two directions is not extreme.

The general expressions will not be set up here; only expressions for slabs without reinforcement $(\Phi = \Phi' = 0)$ will be shown. Expressions with the reinforcement included can be seen in Andreasen [25].

Slabs with a short side without horizontal restraints are treated first. The total load carrying capacity is given by

$$\frac{P}{h^2 f_c} = \frac{12\,S}{h^2(3 - \frac{x+y}{b})} \tag{31}$$

The parameter S is given by

for $\delta \leq \frac{2}{2+\kappa_z}$

$$S = \frac{\nu}{4} h^2 (2\frac{b}{a} + \frac{1}{2}\frac{a}{x}) [1 - (2-\kappa_z)\delta + \frac{1}{12}(12 - 4\kappa_z - 3\kappa_z{}^2)\delta^2] \tag{32}$$

and for $\delta > \frac{2}{2+\kappa_z}$

$$S = \frac{\nu}{6} h^2 (2\frac{b}{a} + \frac{1}{2}\frac{a}{x}) [2\delta_z'^2 + 6\delta_z' + 3 - 2\sqrt{\delta_z'(\delta_z' + 2)^3}] \tag{33}$$

where

$$\kappa_z = \frac{\frac{1}{2}\frac{a}{x} + 2\frac{x}{a} + 2\frac{y}{a}}{2\frac{b}{a} + \frac{1}{2}\frac{a}{x}} \tag{34}$$

$$\delta_z' = \frac{\delta}{\kappa_z} \tag{35}$$

For the slabs with a long side without horizontal restraint, the expressions are

$$\frac{P}{h^2 f_c} = \frac{12\,S}{h^2(3 - 2\frac{v}{b})} \tag{36}$$

S is given by

for $\delta \leq \frac{2}{2+\kappa_v}$

$$S = \frac{\nu}{4} h^2 (\frac{a}{v} + \frac{1}{2}\frac{b}{a-z}) [1 - (2-\kappa_v)\delta + \frac{1}{12}(12 - 4\kappa_v - 3\kappa_v^2)\delta^2] \tag{37}$$

and for $\delta > \frac{2}{2+\kappa_v}$

$$S = \frac{\nu}{6} h^2 \left(\frac{a}{v} + \frac{1}{2}\frac{b}{a-z}\right)\left[2\delta_v'^2 + 6\delta_v' + 3 - 2\sqrt{\delta_v'(\delta_v' + 2)^3}\right] \qquad (38)$$

where

$$\kappa_v = \frac{\frac{a}{v} + \frac{v}{a-z}}{\frac{a}{v} + \frac{1}{2}\frac{b}{a-z}} \qquad (39)$$

$$\delta_v' = \frac{\delta}{\kappa_v} \qquad (40)$$

Considering the expressions (32) and (33) and the expressions (37) and (38), it is noted that they are similar to expressions (25) and (28), respectively, which are valid for the rectangular slab with horizontal restraints on all edges.

Expressions (31) - (40) contain the variable parameters. For the slab with horizontal restraints on all four edges, satisfactory results can be obtained by using the value for the variable parameters found by a normal yield line calculation. For the slab with horizontal restraints on three sides only, it is also possible to do this. A slab with $\Phi = \Phi' = 0$ can not be treated by a normal yield line calculation. The value for the parameters can in this case be estimated by considering a slab with $\Phi = \Phi'$, clamped at the three edges with horizontal restraints and simply supported at the side without horizontal restraint.

4.5 COMPARISON WITH OTHER EXPRESSIONS FOR THE LOAD CARRYING CAPACITY

Many theoretical analyses have been carried out to solve the membrane action problem in reinforced concrete slabs. In some of them the same assumptions are used as in this paper. A comparison between the expressions given in the literature and the expressions given here has been made.

In general, the expressions set up by other authors do not contain the effectiveness factor ν (assumed to be 1), and the expressions are not immediately valid for rectangular slabs.

Expressions from Janas [14], Morley [15], Bræstrup [19], Bræstrup & Morley [20], Christiansen [21] and Eyre & Kemp [23] can be compared with the expressions in this paper. Full agreement is found in the special cases where the expressions from the literature are valid, see Andreasen [25].

5. THEORY COMPARED WITH TEST RESULTS

The theoretical expressions for the load carrying capacity from the previous section will now be compared with test results.

Many tests have been carried out to illustrate the compression membrane effect in reinforced concrete slabs. Many of these tests are collected in Christiansen & Frederiksen [27] and [28]. The test results in these two papers are treated empirically. The result obtained is that the total load carrying capacity for a slab with compression membrane action can be written

$$P_T = P_J + 1.0\ h^2 f_c \qquad (41)$$

$$P_T = P_J + 0.7\ h^2 f_c \qquad (42)$$

for slabs with uniform load and horizontal restraints on 4 and 3 sides, respectively. P_J is the normal yield line load carrying capacity (the Johansen load), and the term $\Omega \cdot h^2 f_c$ (Ω equal to 1.0 or 0.7) is a measure for the load carrying capacity from the membrane action. These expressions are very simple in comparison to the expressions presented here and in other papers. If (41) and (42) are compared with (22) it is seen that the terms with m and m_m in (22) correspond to P_J and $1.0\ h^2 f_c$ in (41), respectively. The reduction caused by the combined bending and membrane action, the term m_n in (22), is not included in (41) and (42).

The expressions from the previous section can be used to find simple expressions for the load carrying capacity, which are theoretically founded and in agreement with the empirical expressions (41) and (42).

5.1 HORIZONTAL RESTRAINTS ON FOUR SIDES

In this section test results on slabs with horizontal restraints on all edges will be treated. The tests are divided into two groups: slabs with normal horizontal restraints and slabs with rigid horizontal restraints.

The tests for slabs with normal restraints are treated first. The data used in the experiments and the test results is shown in table 1. A more detailed description of the tests can be found in the relevant papers or in Christiansen & Frederiksen [27] and Andreasen [25].

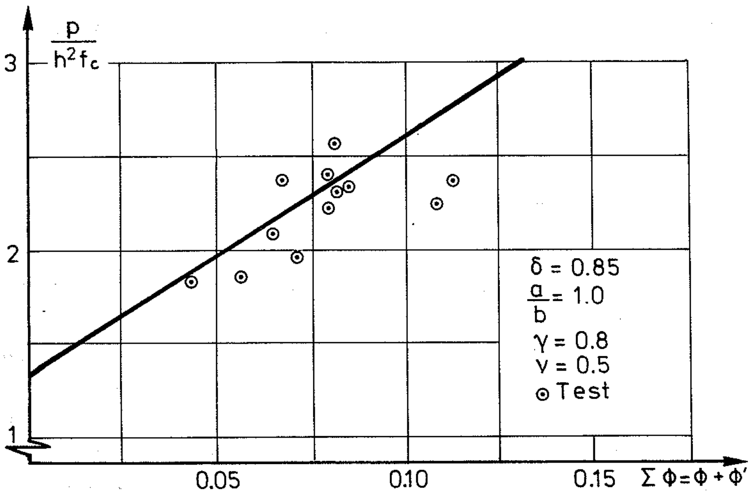

Figure 10: Test results and theoretical load carrying capacity from Hung & Navy's tests (see table 1).

Author	Slab mark	Dimensions a x b x h [mm]	Slenderness $\lambda = \frac{a+b}{2h}$	Compress. concrete strength f_c [Mpa]	Relative effective depth $\gamma = \frac{d}{h}$	$\gamma' = \frac{d'}{h}$	Reinforcement degree $10^3\Phi$	$10^3\Phi'$	Relative deflection $\delta = \frac{w}{h}$	Test load $\frac{P_{test}}{h^2 f_c}$
Hopkins & Park [29]	1	1588x1588x49	32.4	30.0	0.75	0.75	18	20	1.030	1.43
Hung & Navy [30]	C1-1	1524x1524x63.5	24.0	38.5	0.80	0.80	57	57	0.888	2.34
	2			38.5			36	36	0.812	1.94
	3			33.1			32	32	0.880	2.11
	4			38.5			25	25	0.820	1.81
	5			34.4			40	40	0.816	2.23
	6			34.4			42	42	0.856	2.34
	7			39.0			34	34	0.816	2.38
	C4-1	1067x1524x63.5	20.4	33.0			40	40	0.744	2.59
	2			39.8			27	27	0.648	1.88
	3			39.8			55	55	0.624	2.24
	4			34.6			40	40	0.680	2.42
	5			34.6			41	41	0.680	2.34
Christiansen & Frederiksen [27]	SØ 1	3640x3650x119	30.6	27.7	0.83	0.71	26	13	0.800	1.91
	SR 1	1250x1250x39	32.1	14.8	0.80	0.67	117	58	1.410	4.15
	SR 2	39	32.1	18.4	0.82	0.69	47	23	1.256	2.69
	SU 3	38	32.9	16.3	-	-	0	0	0.895	1.22
	LR 4	1000x1250x41	27.4	14.3	0.81	0.68	57	57	0.756	3.73
	LU 5	39	28.8	15.5	-	-	0	0	0.846	1.83
	LR 6	1000x1500x40	31.3	14.4	0.88	0.83	58	58	0.800	3.52
	LR 7	39	32.1	13.6	0.80	0.74	63	63	0.923	3.73
	LU 8	47	26.6	15.5	-	-	0	0	0.447	1.31
	LR 9	1000x1750x41	33.5	15.1	0.83	0.68	54	54	0.902	3.48
	LU10	46	29.9	14.0	-	-	0	0	0.913	1.59
	LR11	1000x2000x40	37.5	13.5	0.80	0.78	62	62	1.025	3.73

Tabel 1: Test data and test results for slabs with normal horizontal restraints on 4 edges.

Hung & Navy's [30] test results together with the theoretical load carrying capacity curve are shown in figure 10. The total dimensionsless load, $P/h^2 f_c$, is shown as a function of the sum $\Phi + \Phi'$. As it is seen from figure 7 and 8 the difference between the load carrying capacity for different values of a/b is small, which is why only one curve valid for a/b = 1 is shown in figure 10. The value used for the relative deflection, δ, is the mean for all the test values in table 1.

The theoretical expressions (22) - (28) for the load carrying capacity can be compared with the test results from table 1. Doing this the mean and the standard deviations shown in table 2 are obtained.

	Correct expression ν from (48) $\frac{P_{test}}{P_J + P_N + P_M}$		Modified expression ν from (49) $\frac{P_{test}}{P_J + P_M}$		
	Real δ	Average δ (0.851)	Real δ	Average δ (0.851)	const. $m_m/\frac{1}{4}\nu$ (= 0.430)
Mean	0.998	1.011	1.000	1.008	1.004
Standard deviation	0.144	0.124	0.144	0.117	0.117

Table 2: Means and standard deviations of the ratio test/theory for the test results from table 1.

The symbols from table 2 are defined as

$$\frac{P_J}{h^2 f_c} = k \cdot m \tag{43}$$

$$\frac{P_N}{h^2 f_c} = k \cdot m_n \tag{44}$$

$$\frac{P_M}{h^2 f_c} = k \cdot m_m \tag{45}$$

where

$$k = \frac{24}{\frac{a}{b}\left(\sqrt{3 + (\frac{a}{b})^2} - \frac{a}{b}\right)^2} \qquad (46)$$

The total theoretical load carrying capacity is then

$$P_{theory} = P_J + P_N + P_M \qquad (47)$$

P_{test} is the total load obtained in the tests, see table 1.
The ratio of test/theory for the different tests in table 1 is shown in Appendix 1.

The theoretical strengths are determined by use of the effectiveness factors

$$\nu = \frac{2.6}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \qquad (48)$$

$$\nu = \frac{2.0}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \qquad (49)$$

Expression (49) is used for the calculation of the modified theoretical strength, see below.

As it can be seen from figure 7, the load carrying capacity varies only marginally for different values of δ, when δ is between 0.7 and 1.6. Therefore an analysis is carried out using an average value of δ for the tests in table 1. The result is also shown in table 2.

As the theoretical expression is complicated, an attempt to simplify it has been made. Comparing the empirical formulas (41) and (42) with test results gives fairly good agreement, see Christiansen & Frederiksen [27] and [28]. The theoretical expression is therefore modified in such a way that the empirical and the theoretical expressions have terms which stand for the same type of contribution to the total load carrying capacity.

The expression for the theoretical load can then be given by

$$P_{theory} = P_J + P_M \qquad (50)$$

The results from the analysis with this expression for the theoretical load carrying capacity are shown in table 2.

Not included in (50) is the term which takes into account the combined bending and membrane action. Thus the effectiveness factor ν is scaled down in proportion to the value in the theoretically correct analysis. P_N is therefore included in the reduced ν value.

The theoretical load carrying capacity varies with the relative deflection, δ , but this variation is small for large values of δ . The variation is mainly due to the variation of the membrane moments with δ , which is why the membrane moments variation must be small for large value of δ . Therefore an analysis is carried out where the membrane load carrying capacity is calculated according to

$$m_m = \frac{1}{4}\nu \cdot g(\delta, x/a) \qquad (51)$$

and where $g(\delta, x/a)$ is taken as a constant. The result of this analysis is also shown in table 2, in the right hand column.

As can be seen in table 2 the results for the five analyses are nearly identical. The result from the calculation with (50) and a constant value for $(m_m/\frac{1}{4}\nu) = g(\delta,x/a)$ is quite good, and is the simplest of the five. Moreover it can easily be used for loads other than uniform load.

The slabs in table 1 are all isotropically reinforced and therefore the expressions from the previous section can be used directly. For orthotropic slabs, the theoretically correct expression can not be used, but it is possible to use the modified expressions instead. The variable parameter(s) can be taken as the value(s) obtained by a normal yield line calculation, which is the reason why the extra work for including the contribution from the membrane action is small.

Using (51) to calculate m_m with a constant value for $g(\delta,x/a)$, the total load can be found by replacing the yield moment with the sum of the yield moment and the membrane moment. Calculations can then proceed in the usual way.

Ockleston [5, 6] has carried out two in-situ tests on slabs with orthotropic reinforcement. Using (50), (49) and a constant value for $g(\delta, x/a) = 0.43$, the results are quite satisfactory. Calculation of the membrane contribution with the parameter x determined by (29) also gives good agreement between test results and the modified expressions for the total load carrying capacity.

The slabs treated above are slabs with "normal horizontal restraints". Tests have also been carried out on slabs with rigid horizontal restraints on four edges. The data used in the experiments and the test results is shown in table 3. A more detailed description of the tests can be found in the relevant papers or in Christiansen & Frederiksen [27] and Andreasen [25].

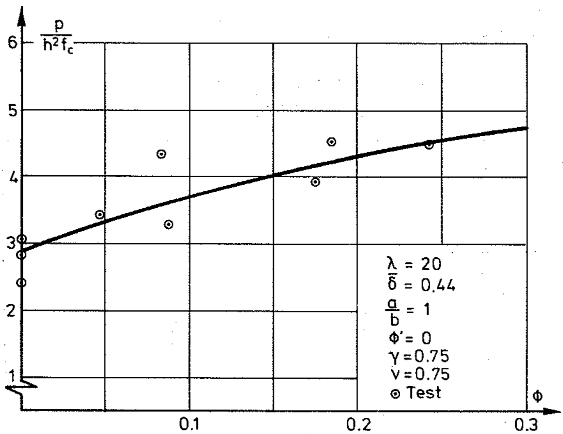

Figure 11: Test results and theoretical load carrying capacity for Brotchie & Holley's [31] *tests with the slenderness ratio* $\lambda = 20$ *, see table 3.*

Some of Brotchie & Holley's [31] test results together with a theoretical load carrying capacity curve are shown in figure 11. As it can be seen from the figure, the theory adequately explains the influence of the mechanical reinforcement degree Φ, on the total load carrying capacity $P/h^2 f_c$.

Author	Slab mark	Dimensions a x b x h [mm]	Slender-ness $\lambda = \frac{a+b}{2h}$	Compr. concr. streng. f_c [MPa]	Rel.effec. depth $\gamma = \frac{d}{h}$	$\gamma' = \frac{d'}{h}$	Reinforcem. degree $10^3\Phi$	$10^3\Phi$	Relative deflect. $\delta = \frac{w}{h}$	Test load $\frac{P_{test}}{h^2 f_c}$
Wood [8]	FS 12	1727x1727x57	30.3	32.5	0.804	-	15	0	0.500	3.31
	FS 13			26.5	0.804	0.804	19	19	0.500	3.28
	FS 14			28.6	-	-	0	0	0.500	2.31
Park [10]	D 1	522x914x32,7	29.0	34.5	-	-	0	0	0.520	2.95
	D 2			34.2	-	-	0	0	0.490	2.77
	D 3			35.5	-	-	0	0	0.390	2.24
	D 4			30.6	-	-	0	0	0.580	2.19
	D 5			24.5	-	-	0	0	0.490	2.66
Brotchie & Holley [31]	32	381x381x19.1	20.0	32.6	-	-	0	0	0.372	3.03
	33			32.6	0.747	-	48	0	0.389	3.40
	34			34.8		-	89	0	0.415	3.29
	35			34.8		-	178	0	0.390	3.96
	36			38.2		-	243	0	0.495	4.50
	42			34.9	-	-	0	0	0.488	2.82
	44			29.9	-	-	0	0	0.362	2.40
	46			37.9	0.747	-	82	0	0.433	4.33
	48			33.6		-	184	0	0.567	4.52
	27	381x381x38.1	10.0	29.0	0.813	-	0	0	0.131	3.66
	28			32.9		-	47	0	0.161	3.98
	29			30.9		-	100	0	0.180	5.04
	31			25.0		-	247	0	0.223	7.15
	30			31.1		-	297	0	0.218	6.12
	45			29.5	-	-	0	0	0.111	3.37
	47			31.5	0.813	-	98	0	0.107	4.58
	49			31.8		-	194	0	0.103	4.76
	37	381x381x76.2	5.0	25.8	-	-	0	0	0.087	7.16
	38			25.8	0.863	-	61	0	0.057	7.15
	39			28.7		-	110	0	0.101	6.94
	40			38.7		-	220	0	0.145	7.88
	41			23.3		-	406	0	0.229	11.99

Table 3: Test data and test results for slabs with rigid horizontal restraints on 4 edges.

	Correct expression ν from (52) $\frac{P_{test}}{P_J + P_N + P_M}$			Modified expression ν from (53) $\frac{P_{test}}{P_J + P_M}$			
	Real δ *	Real δ **	Average δ ** (= 0.457)	Real δ *	Real δ **	Average δ ** (= 0.457)	const. $\frac{m_m}{\frac{1}{4}\nu}$ ** (= 0.640)
Mean	1.007	1.007	1.008	1.001	1.036	1.037	1.005
Standard deviation	0.145	0.152	0.153	0.153	0.163	0.165	0.158

* without tests with λ = 5
** without tests with λ = 5 and λ = 10

Table 4: Means and standard deviation on the ratio test/theory for the test results in table 3.

Comparing the theoretical and the modified theoretical expressions with the test results from table 3, the results shown in table 4 are obtained. The ratio test/theory for the different tests are shown in Appendix 2.

It can be shown that the value of the slenderness ratio λ, for $\lambda \lesssim 20$, has a large influence on the value of δ at maximum load. This can also be seen in table 3, where smaller values of λ have resulted in smaller values of δ. Therefore the analysis with constant values of δ and $g(\delta, x/a)$ is only carried out on the tests with $\lambda \gtrsim 20$. Moreover the tests of Brotchie & Holley with λ = 5 are not included in any of the analysis because the slenderness is so small that it is questionable whether the normal slab theory is valid for these slabs.

The theoretical and the modified theoretical load carrying capacities are calculated from (43) - (47) and (50) where the effectiveness factor is determined by use of

$$\nu = \frac{4.15}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \qquad (52)$$

$$\nu = \frac{3.60}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \qquad (53)$$

Equation (53) is used for the calculation of the modified theoretical strength.

The results obtained with the modified theoretical expression and constant $g(\delta, x/a) = m_m/\frac{1}{4}\nu$ are quite satisfactory, taking into account the small relative deflection, δ, and the greater influence on the load carrying capacity which follows from that (see table 4).

Park [10] and Powell [32] have also carried out tests. Test data and test results for these experiments are shown in table 5.

Park's slabs are orthotropically reinforced. Since the ratio between the ultimate positive and negative moment in the two directions is nearly the same, the affinity theorem can be used. The relative deflection, δ, is not given in Powell's tests.

Comparing the tests with the modified theoretical expression for the strength, the results shown in table 6 are obtained. The ratio test/theory for the different tests is shown in Appendix 3.

$\frac{P_{test}}{P_J + P_M}$ ν from (53)	Park			Powell	
	Real δ	Average δ (= 0.46)	const. $\frac{m_m}{\frac{1}{4}\nu}$ (= 0.64)	Average δ (= 0.46)	const. $\frac{m_m}{\frac{1}{4}\nu}$ (= 0.64)
Mean	1.01	1.01	0.94	1.19	1.09
Standard deviation	0.17	0.13	0.12	0.17	0.14

Table 6: Means and standard deviation on the ratio test/theory for the test results in table 5.

As it is seen from the results in table 6, the correspondance between tests and theory is satisfactory for these tests also.

The results presented in this section indicate that the membrane moment can be considered to be independent of the relative deflection and the yield line pattern. The membrane action can then be included by adding the membrane moment to the normal yield line moment.

Author	Slab mark	Dimensions a x b x h [mm]	Slen-dern.	Compr. concr. streng. [MPa]	Rel.effec. depth $\gamma = \frac{d}{h}$	Rel.effec. depth $\gamma' = \frac{d'}{h}$	Reinforcement degree Short span $10^3\phi$	Short span $10^3\phi'$	Long span $10^3\phi$	Long span $10^3\phi'$	Rel. deflec. $\delta = \frac{w}{h}$	Test load $\frac{P_{test}}{h^2 f_c}$
G. Park [10]	A1	1016x1524x50.8	25.0	32.9	0.85	0.78	17	34	17	34	0.50	3.78
	A2			29.5	0.83	0.75	33	65	18	36	0.50	4.39
	A3			34.5	0.82	0.72	60	121	16	33	0.45	4.53
	A4			27.7	0.30	0.69	108	217	20	41	0.37	5.57
G. Powell [32]	46	522x914x32.7	22.0	40.0	0.8	-	10	10	10	10	-*	3.47
	47			44.8			9	9	9	0	-	2.66
	50			37.2			19	19	19	19	-	3.99
	54			41.0			27	27	27	27	-	3.99
	55			36.8			30	30	30	30	-	4.61
	58			40.0			46	46	46	46	-	3.83
	59			39.3			47	47	47	47	-	3.99
	62			41.0			70	70	70	70	-	4.65
	63			36.4			80	80	80	80	-	5.72
	48			41.0	-		0	0	0	0	-	2.78
	53			37.6			0	0	0	0	-	3.45
	56			38.2			0	0	0	0	-	3.03
	57			39.6			0	0	0	0	-	2.35
	60			39.7			0	0	0	0	-	2.53
	64			39.7			0	0	0	0	-	2.73

* The relative deflection δ is not given.

Table 5: Test data and test results for slabs with rigid horizontal restraints on all edges.

5.2 HORIZONTAL RESTRAINTS ON THREE SIDES

Tests on slabs with normal supports along the edges and normal horizontal restraints on three sides only have also been carried out. The test is reported in Hopkins & Park [29], Park [33], Hung & Nawy [30] and Christiansen & Frederiksen [28]. In Christiansen & Frederiksen the results of the other authors are also shown.

The theoretically correct expression for slabs with horizontal restraints on three sides can be used to calculate the theoretical strength of the slabs. From this analysis it turns out that the same effectiveness factor ν can be used as for the slabs with normal horizontal restraints at all edges, expressions (48) and (49). (48) is used when the strength is determined from the correct expressions and (49) when the modified expression is used.

The standard deviation of the ratio test/theory was about 0.13, both when the theoretical load carrying capacity was calculated with the values of δ from the tests and with an average value of 0.85. The deviation was smaller when the theoretical strength was calculated with the modified expression, than if the theoretically correct expression was used, but the difference was small.

The yield line pattern was determined by the normal expressions for slabs without horizontal restraints. This indicates that the membrane action can be taken into account by determining the yield line pattern in the normal way. Before inserting the optimal values for the variable parameters in the expression for the load carrying capacity, the positive yield moment is replaced by the sum of the positive yield moment and the membrane moment. The membrane moment should first be added to the positive yield moment after the optimal yield line pattern is determined. This is because the membrane action influences the optimal yield line pattern very little, and the membrane moment is of the same order of magnitude as the normal yield moments.

For a more detailed description of the test treatments the reader is referred to Andreasen [25].

5.3 THE RELATIVE DEFLECTION

The relative deflection, δ, is a very important parameter for determinating the membrane moment. As shown in section 5.1, δ can be taken as a constant for slabs with the slenderness ratio $\lambda \gtrsim 20$.

The slabs with rigid horizontal restraints on all edges and $\lambda \geq 20$ have a relative deflection of about 0.45. If this deflection is assumed to be identical for slabs with rigid and normal horizontal restraints, the deflection of slabs with normal horizontal restraints can be estimated in a simple way. The total relative deflection is determined by

$$\delta = \delta_o + \delta_s \qquad (54)$$

where δ_o is the deflection from the flexibility of the slab and δ_s is the deflection from the horizontal displacement of the edges. δ_o is in the order of 0.4-0.5, and can be considered as being the deflection for the rigid horizontal restraints slabs. δ_s can be found by considering figure 12.

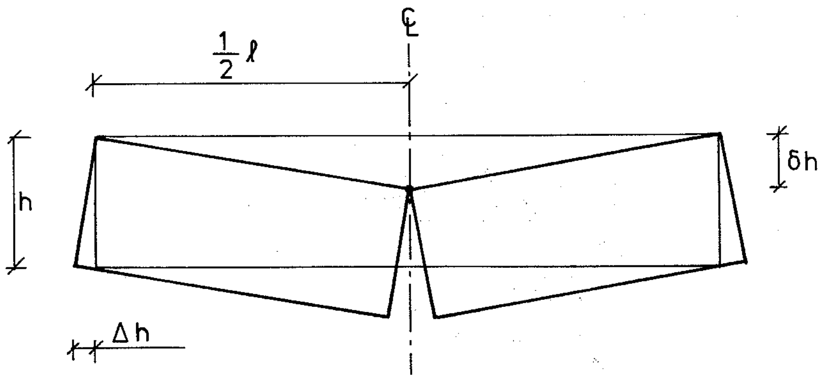

Figure 12: Slab part with deflection δh *in the middle and a horizontal displacement* Δh *at the edges.*

The connection between the horizontal displacement, Δh, and the vertical deflection, δh, can be found to be

$$\delta_s = \delta = \Delta \cdot \frac{1}{2}\frac{\ell}{h} \qquad (55)$$

The horizontal displacement, Δh, can be determined in the normal way. The stresses on the edges from the membrane action are applied on the surroundings.

This calculation can be used to check the surroundings, and to make a control of the size of the total relative deflection δ.

5.4 LOAD-DEFLECTION CURVE: THEORETICAL AND EXPERIMENTAL

The theoretical load-deflection relation from the expressions in section 4 will not be the same as the load-deflection curve in a test. The theoretical load is decreasing for increasing deflection (until a certain limit). The experimental load is first increased from zero, is then decreasing and perhaps increasing again, depending on the reinforcement degree and how far the test is continued.

In figure 13, two examples of theoretical and experimental load-deflection relations are shown.

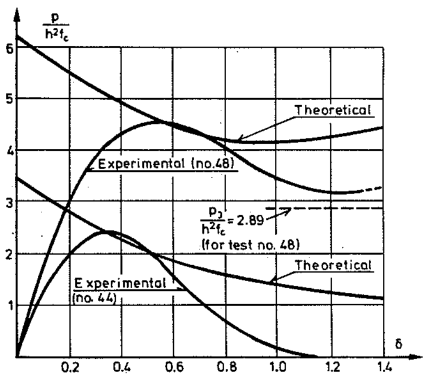

Figure 13: Theoretical and experimental load-deflection relation for two of Brotchie & Holley's [31] slabs. The theoretical curve is determined by use of parameters from the test and a value of ν *which bring the theoretical and experimental curves to intersect at the experimental maximum point.*

As can be seen from the figure there is agreement between the theoretical and experimental curve in only a small area. This is because the theoretical curve is determined by using the same assumptions for all values of δ and there will only be agreement between theory and test in a small interval.

The same situation is also found in other cases, where the strength is strongly influenced by the concrete strength. Examples are pure bending in a overreinforced section and shear in beams.

The experimental load-deflection curve for the unreinforced slab no. 44 must be noticed. The relation is similar (has the same form) to those found in normal compression concrete strength tests. Moreover, the experimental load-deflection curve after maximum load for slab no. 48 must be observed. The minimum load is only slightly greater than the Johansen load.

6. DESIGN RECOMMENDATIONS

The theoretical expressions from section 4 can be used, but they are complicated. As shown in section 5, the agreement obtained between tests and theory is found to be quite good, even if the theoretical expressions are modified. It appears from the analysis that the membrane action can be taken into account by adding a membrane moment, m_M, to the normal positive bending moment, m_J. The normal negative bending moment, m_J', is unchanged. The moments are given by

$$m_{JM} = m_J + m_M \tag{56}$$

$$m_J = \Phi\left(\frac{d}{h} - \frac{\Phi}{2\nu}\right) \cdot h^2 f_c \tag{57}$$

$$m_M = \frac{1}{4} h^2 \nu f_c \cdot g(\delta, x/a) \tag{58}$$

$$m_J' = \Phi'\left(\frac{d'}{h} - \frac{\Phi'}{2\nu}\right) \cdot h^2 f_c \tag{59}$$

where the effectiveness factor ν and the function g can be determined by

$$\nu = \frac{2.0}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \tag{60}$$

$$g(\delta,\ x/a) = 0.43 \tag{61}$$

The above expressions are valid for slabs with normal horizontal restraints at the edges. They have been tested on slabs with ratios of side length (b/a) between 1 and 2, the sum of the mechanical reinforcement degrees $\Phi + \Phi'$ less than 0.18, the concrete compression cylinder strength between 13 and 45 MPa and the slenderness $\lambda = (a + b)/2h$ in the interval 20 to 40.

The calculations are carried out as a normal yield line calculation. The yield line pattern is determined for a slab without membrane action. When the yield line pattern is fixed the positive bending moment, m_J, is replaced by the sum of the positive bending moment, m_J and the membrane moment, m_M, $m_{JM} = m_J + m_M$. The sum m_{JM} is used everywhere in the normal expression for the load carrying capacity where m_J is placed.

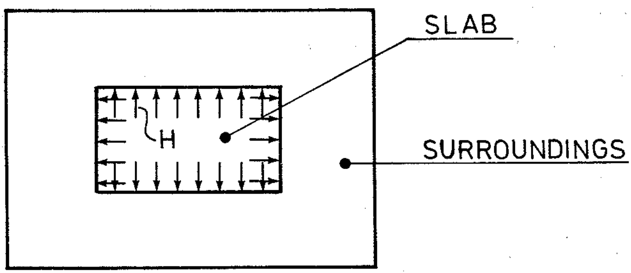

Figure 14: Forces on surroundings from a slab with membrane action.

The surroundings must be controlled for the horizontal forces from the slab, see figure 14. The horizontal forces can be determined by

$$H = \xi h \cdot \nu f_c \tag{62}$$

where the relative compression zone, ξ , can be found from

$$\xi = 1 + \delta' - \sqrt{\delta'(\delta' + 2)} \tag{63}$$

The modified relative deflection, δ' , is defined in section 4.

The expression for ξ (63) is not correct, but it is on the safe side when $\Phi > \Phi'$.

7. CONCLUSION

In many cases the theory of plasticity has given quite a good description of the ultimate strength of concrete structures, in spite of the limited concrete ductility . Using a rigid plastic material model for concrete, modification factors must be used for taking into account the lacking plasticity. The membrane action in reinforced concrete slabs is treated by this model and encouraging results are obtained.

Expressions for the ultimate strength of rectangular reinforced concrete slabs with horizontal restraints at the edges are set up. The theoretical expressions are found to be satisfactory. Since the expressions are too complicated for practical use, they are simplified. The modified expressions are also compared with tests and the agreement is quite good. It turns out that the membrane action can be taken into account by adding a membrane moment to the normal positive bending moment and the sum is then taken as the positive moment. The negative bending moment is unchanged. The calculations are then carried out as a normal yield line calculation.

The test do not yet cover slabs with concentrated loads; for instance wheel loads on bridge decks. Before applying the theory to such problems further test should be performed.

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APPENDIX 1

THE RATIO TEST/THEORY FOR SLABS WITH NORMAL HORIZONTAL RESTRAINTS ON ALL EDGES

The ratio test/theory for the different tests in table 1 are shown in the table below. The mean values and standard deviations are also shown in table 2.

Slab mark	Correct expression ν from (48) $\frac{P_{test}}{P_J + P_N + P_M}$		Modified expression ν from (49) $\frac{P_{test}}{P_J + P_M}$		
	Real δ	Average δ (= 0.851)	Real δ	Average δ (= 0.851)	const. $m_m/\frac{1}{4}\nu$ (= 0.430)
1	0.897	0.842	0.934	0.870	0.892
1C-1	0.926	0.925	0.838	0.831	0.842
1C-2	0.956	0.963	0.890	0.901	0.916
1C-3	1.060	1.053	1.017	1.008	1.027
1C-4	1.029	1.038	1.005	1.016	1.037
1C-5	1.017	1.024	0.943	0.953	0.968
1C-6	1.052	1.051	0.971	0.972	0.988
1C-7	1.207	1.216	1.132	1.144	1.164
4C-1	1.124	1.145	1.044	1.078	1.065
4C-2	0.977	1.042	0.937	1.013	0.998
4C-3	0.858	0.875	0.754	0.800	0.792
4C-4	1.048	1.083	0.964	1.018	1.005
4C-5	1.002	1.035	0.920	0.970	0.958
SØ 1	1.039	1.062	1.076	1.101	1.128
SR 1	1.058	1.102	1.096	0.992	1.006
SR 2	1.180	1.097	1.218	1.080	1.103
SU 3	0.726	0.704	0.944	0.915	0.955
LR 4	1.170	1.195	1.091	1.125	1.125
LU 5	1.051	1.055	1.366	1.371	1.371
LR 6	0.987	0.995	0.931	0.945	0.928
LR 7	1.081	1.072	1.029	1.009	0.990
LU 8	0.521	0.760	0.678	0.988	0.943
LR 9	1.087	1.080	1.046	1.030	0.990
LU10	0.917	0.875	1.192	1.137	1.033
LR11	0.986	0.978	0.973	0.932	0.883
Mean	0.998	1.011	1.000	1.008	1.004
Stand. devia.	0.144	0.124	0.144	0.117	0.117

APPENDIX 2

THE RATIO TEST/THEORY FOR SLABS WITH RIGID
HORIZONTAL RESTRAINTS ON ALL EDGES

The ratio test/theory for the different tests in table 3 are shown in the table below. The means and standard deviations are also shown in table 4.

Slab mark	Correct expression ν from (52) $\frac{P_{test}}{P_J + P_N + P_M}$			Modified expression ν from (53) $\frac{P_{test}}{P_J + P_M}$			
	Real δ	Real δ	Aver. δ (= 0.457	Real δ	Real δ	Aver. δ (= 0.457)	const. $\frac{m_m}{\frac{1}{4}\nu}$ (= 0.640)
FS12	1.186	1.186	1.141	1.286	1.286	1.239	1.221
FS13	0.990	0.990	0.957	1.007	1.007	0.975	0.963
FS14	0.821	0.821	0.787	0.947	0.947	0.908	0.894
D1	1.064	1.064	1.109	1.227	1.227	1.279	1.182
D2	1.075	1.075	1.037	1.240	1.240	1.195	1.105
D3	0.792	0.792	0.854	0.914	0.914	0.985	0.910
D4	0.884	0.884	0.776	1.019	1.019	0.894	0.826
D5	0.874	0.874	0.843	1.007	1.007	0.972	0.898
32	1.013	1.013	1.103	1.168	1.168	1.271	1.251
33	1.025	1.025	1.081	1.010	1.010	1.062	1.050
34	0.953	0.953	0.978	0.856	0.856	0.878	0.870
35	0.979	0.979	1.007	0.787	0.787	0.812	0.806
36	1.103	1.103	1.093	0.855	0.855	0.842	0.837
42	1.095	1.095	1.062	1.262	1.262	1.224	1.205
44	0.761	0.761	0.837	0.877	0.877	0.964	0.949
46	1.333	1.333	1.353	1.204	1.204	1.222	1.210
48	1.168	1.168	1.125	0.950	0.950	0.905	0.899
27	0.903	-	-	1.041	-	-	-
28	0.978	-	-	0.977	-	-	-
29	1.117	-	-	1.007	-	-	-
31	1.251	-	-	0.986	-	-	-
30	1.115	-	-	0.843	-	-	-
45	0.822	-	-	0.948	-	-	-
47	0.974	-	-	0.883	-	-	-
49	0.907	-	-	0.729	-	-	-
Mean	1.007	1.007	1.008	1.001	1.036	1.037	1.005
Stand. devia.	0.145	0.152	0.153	0.153	0.163	0.165	0.158

APPENDIX 3

THE RATIO TEST/THEORY FOR SLABS WITH RIGID HORIZONTAL RESTRAINTS ON ALL EDGES

The ratio test/theory for the different tests in table 5 are shown in the table below. The means and standard deviations are also shown in table 6.

$\frac{P_{test}}{P_J + P_M}$ ν from (53)	Park				Powell		
	Slab mark	Real δ	Average δ (= 0.46)	const. $\frac{m_m}{\frac{1}{4}\nu}$ (= 0.64)	Slab mark	Average δ (= 0.46)	const. $\frac{m_m}{\frac{1}{4}\nu}$ (= 0.64)
	A1	1.173	1.135	1.069	46	1.331	1.212
	A2	1.117	1.084	1.005	47	1.088	0.990
	A3	0.956	0.958	0.890	50	1.303	1.199
	A4	0.808	0.848	0.795	54	1.217	1.130
					55	1.310	1.217
					58	0.945	0.889
					59	0.970	0.913
					62	0.945	0.899
					63	1.054	1.004
					48	1.290	1.154
					53	1.533	1.371
					56	1.357	1.214
					57	1.071	0.959
					60	1.155	1.033
					64	1.246	1.115
Mean		1.014	1.006	0.940		1.188	1.087
Stand. deviation		0.165	0.129	0.122		0.173	0.144

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