Afdelingen for Bærende Konstruktioner
Department of Structural Engineering
Danmarks Tekniske Højskole · Technical University of Denmark

ULTIMATE LOAD CARRYING CAPACITY OF UNBONDED PRESTRESSED REINFORCED CONCRETE BEAMS

CHEN GANWEI
M.P. NIELSEN
K. JANOS

Serie R No 259 1990

ULTIMATE LOAD CARRYING CAPACITY OF UNBONDED PRESTRESSED REINFORCED CONCRETE BEAMS

CHEN GANWEI
M.P. NIELSEN
K. JANOS

Ultimate Load Carrying Capacity of Unbonded Prestressed Reinforced Concrete Beams

Tryk:
Afdelingen for Bærende Konstruktioner
Danmarks Tekniske Højskole
Lyngby
ISBN 87-7740-048-8

Preface

This work is a part of the work carried out by Chen Ganwei in connection with his Ph.D study. Prof. M. P. Nielsen acted as supervisor. Mr. K. Janos collected the test data and performed some preliminary calculations of the bending strength for comparison with different code formulas.

The work was supported by AEC Consulting Engineers Ltd.

Resume

I denne afhandling undersøges anvendelsen af plasticitetsteorien på forspændte bjælker med ikke injicerede kabler.

Det er tidligere fundet, at man ved bøjningsundersøgelser ikke kan opnå udnyttelse af spændarmeringen i samme grad som ved forspændte bjælker med injicerede kabler. Dette bekræftes af den teoretiske undersøgelse efter den plasticitets teoretiske model og det skyldes, at trykzonehøjden ved overgang til overarmeret tværsnit kun er ca. halvt så stor som ved bøjning af bjælker med injicerede kabler.

Forskydningsbæreevnen af bjælker med ikke–injicerede kabler er ligeledes undersøgt på basis af plasticitetsteorien. Det viser sig, at de sædvanlige udtryk kan bruges, når blot den effektive plastiske trykstyrke undertiden regnes lidt lavere.

Summary

In this report the plastic theory for reinforced concrete structure is investigated in relation to beams with unbonded tendons.

Concerning the bending resistance of beams with unbonded tendons, it has been found previously, that the stress at failure in the prestressing bars is often lower than in a correspending beams with bonded tendons. This is confirmed by the plastic analysis and according to this, the phenomenon is due to the fact, that the transition from the normally reinforced case to the overreinforced case takes place at about half the compression depth compared to a beam with bonded tendons.

The shear resistance of beams with unbonded tendons can be treated by the plastic theory if in some cases the value of the effectiveness factor is put at a lower value than for beams with bonded tendons.

List of symbols

A_c : area of cross–section.
A_p : cross–sectional area of prestressing reinforcement.
A_s : cross–sectional area of horizontal reinforcement.
A_{swv} : cross–sectional area of vertical web reinforcement per unit length.

b : web width of beam.

d : effective depth of beam.
d_p : distance from extreme compression fiber to centroid of unbonded.tendons.

$f_{0.2}$: proof–stress of unbonded tendons.
f_c : uniaxial compressive strength of concrete.
f_c^* : plastic compressive strength of concrete, defined ad $f_c^* = \nu f_c$.
f_{cu} : compressive cube strength of concrete.
f_p : yield strength of prestressing reinforcement.
f_{pu} : ultimate strength of unbonded tendons.
f_{se} : effective prestress of unbonded tendons.
F_{se} : effective prestress acting on the section.
F_{su} : tension force of the unbonded tendons at the ultimate load carried by the beam.
f_y : yield strength of horizontal reinforcement.
f_{yw} : yield strength of web reinforcement.

h : depth of cross–section.
h^* : effective shear depth.

l : clear span of beam.
L_o : distance between end anchorages.

M_p : bending yield moment.
m_p : dimensionless bending yield moment, defined as $M_p/(bd_p^2f_c)$.

P : external loads.

V_w : web reinforcement ratio, defined as A_{swv}/b.
V, V_u : ultimate shear force.
V_{cal} : calculated shear carrying capacity.
V_{test} : observed ultimate shear force in tests.

y_o : depth of compressive zone of concrete.

α : angle.

ϵ : strain of reinforcement.
ϵ_1, ϵ_2: principal strains.

ν : effectiveness factor for the compressive strength of concrete.
ν_b : effectiveness factor for bending.

σ : stress of reinforcement. Standard deviation.
σ_1, σ_2: principal stresses.

τ : shear stress.

ϕ_p : horizontal reinforcement degree, defined as $\phi_p = A_p f_{0.2}/(bd_p f_c)$.
ϕ_v : vertical web reinforcement degree, $\phi_v = r_w \cdot f_{yw}/f_c$.
ϕ_v^* : effective vertical web reinforcement degree, $\phi^*_v = \phi_v/\nu$.

List of contents

		Page
1.	Introduction	1
2.	Basic assumptions	1
3.	Bending strength of unbonded prestressed reinforced concrete beams	3
	3.1 Theoretical background	4
	3.2 Test results of unbonded prestressed concrete beams failed in flexure	6
	3.3 Effectiveness factor for bending ν_b	11
	3.4 Comparison of tests with theory	13
	3.5 Conclusion	15
4.	Shear strength of prestressed concrete beams with unbonded tendons	16
	4.1 Theoretical background	16
	4.2 Test results of prestressed beams with unbonded tendons failed in shear	18
	4.3 ν–value for prestressed beams with unbonded tendons without bond	20
	4.4 Comparison of tests with theory	20
	4.5 Conclusion	22
5.	References	23

1. Introduction

In post–tensioned unbonded prestressed concrete the prestressed reinforcement is not bonded to the concrete. Changes in the tensile stresses of the prestressed tendons due to flexure are developed through the displacements of the end anchorages The technique of unbonded prestressed concrete has considerably expanded the range of application of prestressed concrete because of the easiness of field work and the economy by avoiding grouting.

Experimental and analytical studies on unbonded prestressed concrete beams have been carried out for decades.

Since it has been shown that many basic load–carrying problems in the theory of reinforced concrete can be solved by the plastic theory, it is natural to investigate whether the ultimate carrying capacity of unbonded prestressed concrete beams can also be solved by this theory.

This paper only deals with the ultimate bending resistance of unbonded prestressed concrete beams and the ultimate shear capacity of unbonded prestressed concrete beams with shear reinforcement.

2. Basic Assumptions

In the following chapters the plastic theory of reinforced concrete beams is used based on the following assumptions:

a) The unbonded prestressed concrete beam is in a state of plane stress.

b) Any premature failures, such as anchorage failure and bearing failure are prevented by special provisions.

c) The reinforcement is perfectly plastic with a stress–strain relation of the type shown in figure 1, i.e. a stress–strain curve having a yield plateau when the yield stress f_y (or proof stress $f_{0.2}$) is reached.

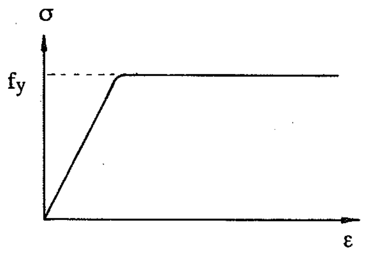


Figure 1 The stress–strain relation of reinforcement.

The reinforcement is assumed to be only capable of carrying longitudinal tensile or compressive stresses, i.e., dowel effects are neglected.

d) The yield condition and constitutive equations for concrete are taken to be the Coulomb's failure criterion with zero tension cut off and the associated flow rule (normality condition), figure 2.

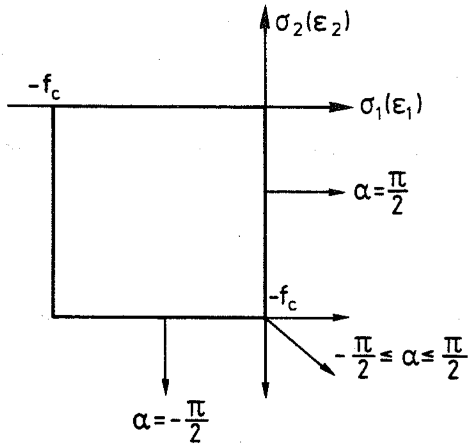


Figure 2 Square yield locus and normality condition for concrete in plane stress.

Since concrete is not a perfectly plastic material, the plastic solutions must be modified by replacing the compressive strength of the concrete f_c by a reduced value νf_c, where $\nu < 1$. The quantity νf_c is called the effective plastic strength of concrete and ν is called the effectiveness factor. The value of ν normally must be determined by experiments.

e) For beams with shear reinforcement, the stirrup spacing is sufficiently small to permit a continuous distribution of the equivalent stirrup forces.

3. Bending Strength of Unbonded Prestressed Reinforced Concrete Beams

The effects of missing bond of the prestressing tendons on the ultimate bending resistance of simply supported rectangular beams has been investigated earlier in some detail.

For calculating the ultimate flexural strength of unbonded prestressed reinforced concrete beams, the current practice in most of codes and standards is to set up an empirical equation for predicting the stress in unbonded prestressed tendons at flexural failure and then use the normal beam theory. Most of the Code methods set out for determination of the ultimate moment of unbonded prestressed concrete beams are simple but inaccurate. Many attempts to create more accurate equations for predicting the stress in the unbonded prestressed tendons at flexural failure by including more influential parameters in the equations have been made. But these alternative methods are in general too tedious to be used in practice.

In this chapter it is demonstrated how the membrane action theory based on the theory of perfectly plastic materials can be developed for unbonded prestressed reinforced concrete beams. The theoretical solution with modification by a simple empirical equation for the effectiveness factor is then compared with the existing test results found in the literature. The agreement is found to be satisfactory.

3.1 Theoretical background

Consider a horizontal, simply supported rectangular concrete beam reinforced with unbonded prestressed tendons and loaded by two symmetrical concentrated loads P, see figure 3.

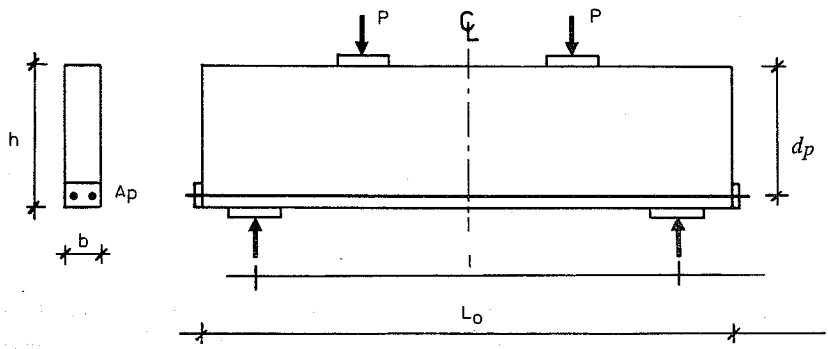
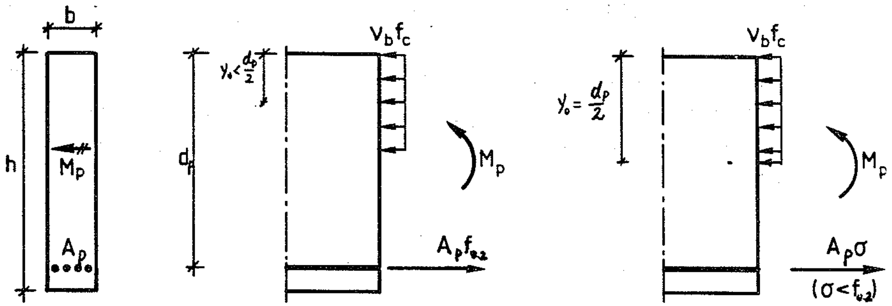


Figure 3 Rectangular unbonded prestressed concrete beam subjected to concentrated loads.

The width and depth of the beam are termed b and h, respectively. The beam is reinforced with the unbonded prestressed tendons with the area A_p and proof stress $f_{0.2}$ along the bottom face of the beam. The quantity l and L_0 denote the clear span and total span, respectively.

It has been observed, that for unbonded post–tensioned beams with no bond between the tendons and the concrete, after cracking, they behave as a shallow tied arch, rather than as a flexural member [71.2].

According to the membrane action (or arch action) analysis, the maximum load is obtained when the depth of the compressive zone of the concrete equals half of the effective depth d_p of the cross section, see figure 4.



a) Normally reinforced section b) Over reinforced section

Figure 4 Normal stress distribution at flexural failure.

For a more detailed presentation, the reader is referred to [88.6].

The complete theoretical solution for unbonded prestressed concrete beams with rectangular section can then be written

$$m_p = \begin{cases} \left[1 - \dfrac{\phi_p}{2\nu_b}\right]\phi_p & \text{for } \phi_p \le \dfrac{\nu_b}{2} \\ \dfrac{3}{8}\nu_b & \text{for } \phi_b > \dfrac{\nu_b}{2} \end{cases} \tag{1}$$

Here we have introduced the dimensionless yield moment $m_p = M_p/(bd_p^2 f_c)$ and the unbonded prestressing reinforcement degree $\phi_p = A_p f_{0.2}/(bd_p f_c)$. In Equation (1) the effectiveness factor ν_b for an unbonded prestressed beam must be determined by test results of flexural failures.

3.2 Test results of unbonded prestressed concrete beams failed in flexure

In this investigation of bending resistance of unbonded prestressed concrete beams, 99 test data for simply supported rectangular beams have been found in the literature. The detailed test data, including the dimensions, the most important material properties and measured ultimate moment capacity of the 99 beams are shown in table 1–8.

In these tables only the ultimate strengths of the prestressing tendons are given. The proof–stress $f_{0.2}$ is taken as 0.875 f_{pu}. In some reports, only the cube strength of concrete is given. In these cases the compressive strength of concrete f_c is taken as 0.8 f_{cu}. Except for [56.2], [69.3] and [81.12], all other test data are collected from [85.1].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING

BEAM MARK	l cm	b cm	h cm	d_p cm	f_c MPa	A_p cm^2	f_{pu} MPa	f_{se} MPa	M_u kNm
A---1	407	15.2	22.8	15.8	34.6	2.31	1550	370	18.9
A---2	407	15.2	22.8	15.5	35.2	2.31	1550	645	23.8
A---3	407	15.2	22.8	15.0	31.9	2.31	1550	1045	31.3
A---4	407	15.2	22.8	14.8	33.8	1.54	1550	1082	22.4
A---5	407	15.2	22.8	14.6	33.8	1.54	1550	672	16.8
A---6	407	15.2	22.8	14.6	32.5	1.54	1550	345	12.7
A---7	407	15.2	22.8	15.2	32.5	3.09	1550	1020	36.2
A---8	407	15.2	22.8	15.3	32.2	3.09	1550	715	28.8
A---9	407	15.2	22.8	15.6	33.4	3.09	1550	343	21.9
A-29R	407	15.2	22.8	17.8	36.8	2.31	1550	985	38.8
A-30R	407	15.2	22.8	17.8	34.1	2.31	1550	326	23.4
A-34R	407	15.2	22.8	15.7	42.9	2.31	1550	1105	35.1
A-35R	407	15.2	22.8	15.6	42.9	1.54	1550	1120	29.3
A-36R	407	15.2	22.8	16.6	35.6	1.16	1550	1110	21.3
A--10	407	22.8	15.2	10.4	35.5	2.31	1550	1050	21.5
A--11	407	22.8	15.2	10.3	33.1	2.31	1550	405	12.2
A--12	407	22.8	15.2	10.2	34.6	1.54	1550	956	14.0
A--13	407	22.8	15.2	10.3	33.8	1.54	1550	418	8.2
A--18	407	22.8	15.2	10.7	33.3	3.09	1550	1038	28.2
A--19	407	22.8	15.2	10.3	33.1	3.09	1550	397	14.4
A--20	407	22.8	15.2	10.0	33.1	1.54	1550	694	11.7
A--23	407	22.8	15.2	10.8	32.6	2.31	1550	715	19.2
A-23R	407	22.8	15.2	10.5	35.8	2.31	1550	745	18.4
A--26	407	22.8	15.2	10.4	32.6	3.09	1550	755	21.5
A--28	407	22.8	15.2	9.9	34.9	0.77	1550	1088	9.0
A-31R	407	22.8	15.2	10.8	32.0	1.95	1550	1062	21.3
A-37R	407	22.8	15.2	7.6	35.6	1.54	1550	1035	10.5
A--14	183	15.2	22.8	15.7	34.7	2.31	1550	1000	33.9
A--15	183	15.2	22.8	16.3	34.7	2.31	1550	419	31.0
A--16	183	15.2	22.8	15.8	31.7	1.54	1550	992	27.0
A--17	183	15.2	22.8	16.1	33.6	1.54	1550	356	19.3
A--21	183	15.2	22.8	15.9	32.0	1.54	1550	580	22.7
A--22	183	15.2	22.8	15.8	31.9	2.31	1550	666	29.7
A--24	183	15.2	22.8	16.1	35.2	3.09	1550	675	38.2
A--25	183	15.2	22.8	16.3	33.8	3.09	1550	380	32.8
A--27	183	15.2	22.8	16.4	31.5	3.09	1550	974	42.0
A-32R	183	15.2	22.8	15.3	38.1	2.31	1550	952	36.4
A-33R	183	15.2	22.8	15.8	39.5	1.54	1550	1015	28.6

Table 1 Testdata from ref. [69.3].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING										
BEAM	l	b	h	d_p	f_c	A_p	f_{pu}	f_{se}	f_{su}	M_u
MARK	cm	cm	cm	cm	MPa	cm^2	MPa	MPa	MPa	kNm
I	500	12.2	33.4	25.7	35.3	5.46	608	427	531	66.4
II	500	12.2	33.4	25.7	15.2	5.28	615	350	460	50.8
III	500	12.0	32.8	25.3	53.8	5.42	626	448	553	74.3
IV	500	12.3	34.1	27.2	18.0	5.40	613	304	423	48.3
VI	500	12.1	33.3	25.9	63.8	2.64	1749	826	1121	74.6

Table 2 Testdata from ref. [59.1].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM	l	b	h	d_p	f_c	A_p	f_{pu}	f_{se}	M_u
MARK	cm	cm	cm	cm	MPa	cm^2	MPa	MPa	kNm
P---1	600	100.0	18.0	14.3	45.8	3.96	2043	1283	90.3
PS--3	360	40.0	18.0	16.2	32.7	0.93	1946	1187	23.6

Table 3 Testdata from ref. [77.12] and [78.5].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM	l	b	h	d_p	f_c	A_p	f_{pu}	f_{se}	M_u
MARK	cm	cm	cm	cm	MPa	cm^2	MPa	MPa	kNm
1	460	35.3	18.0	12.0	30.1	2.79	1766	1163	42.8
2	460	70.5	18.0	12.0	30.1	2.79	1766	1145	45.7
3	460	118.2	18.0	12.0	30.1	1.16	1840	1197	21.9
4	340	35.3	18.0	12.0	34.4	2.79	1766	1163	44.0
5	340	70.5	18.0	12.0	34.4	2.79	1766	1154	47.7
6	340	118.2	18.0	12.0	34.4	1.16	1840	1220	20.8
7	220	35.3	18.0	12.0	30.8	2.79	1766	1164	44.1
8	220	70.5	18.0	12.0	30.8	2.79	1766	1168	47.4
9	220	118.2	18.0	12.0	30.8	1.16	1840	1204	20.6

Table 4 Testdata from ref. [81.12]

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM MARK	l cm	b cm	h cm	d_p cm	f_c MPa	A_p cm^2	f_{pu} MPa	f_{se} MPa	M_u kNm
PS--1	289	33.7	16.6	10.3	56.0	1.00	2063	1050	14.2
PS--2	288	34.2	16.7	10.0	51.0	2.00	2063	850	20.1
PS--3	288	34.2	16.8	7.1	58.0	2.00	2063	1175	17.8
PS--4	429	51.6	24.3	11.6	38.0	3.00	2063	779	35.5
PS--5	433	51.7	24.2	11.1	42.0	3.00	2063	1284	42.1
PS--6	434	51.5	24.2	11.6	47.0	5.00	2063	1057	61.0
PS-13	288	34.0	16.0	7.4	42.5	2.00	2063	1173	16.5
PS-15	432	51.2	24.0	14.8	36.5	2.00	2063	612	36.1
PS-22	479	102.0	48.2	21.0	46.7	15.4	2063	934	376.0
PS-23	288	34.1	16.1	8.0	29.7	2.00	2063	838	14.8

Table 5 Testdata from ref. [81.1] and [83.1].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM MARK	l cm	b cm	h cm	d_p cm	f_c MPa	A_p cm^2	f_{pu} MPa	f_{se} MPa	M_u kNm
2	480	20.0	40.0	35.0	42.3	2.74	1782	1013	144.0
4	480	20.0	40.0	35.0	52.2	2.78	1847	994	107.8
1	480	40.0	20.0	15.5	48.8	2.74	1782	1010	61.8
3	480	40.0	20.0	15.5	49.3	2.78	1847	1022	54.5

Table 6 Testdata from ref. [80.1].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM MARK	l cm	b cm	h cm	d_p cm	f_c MPa	A_p cm^2	f_{pu} MPa	f_{se} MPa	M_u kNm
30128	274	15.2	30.5	21.1	40.7	1.03	1851	758	23.0
30144	274	15.2	30.5	21.1	36.2	1.03	1851	841	25.1
30307	274	15.2	30.5	21.1	40.0	2.06	1851	896	46.1
30428	274	15.2	30.5	21.1	36.5	3.09	1851	827	56.7
31428	274	15.2	30.5	21.1	36.5	3.09	1851	917	59.0

Table 7 Testdata from ref. [56.2].

THE TEST DATA OF UNBONDED PRESTRESSED BEAMS FAILED IN BENDING									
BEAM	l	b	h	d_p	f_c	A_p	f_{pu}	f_{se}	M_u
MARK	cm	cm	cm	cm	MPa	cm^2	MPa	MPa	kNm
0U097	275	15.2	30.0	19.2	22.3	0.96	1568	836	18.3
0U255	275	15.2	30.0	19.6	20.2	2.31	1568	856	33.4
0U024	275	15.2	31.0	19.9	49.8	0.55	1671	851	15.5
0U065	275	15.0	30.7	20.8	36.0	1.11	1671	850	28.6
0U100	275	15.2	30.7	21.2	26.3	1.30	1671	866	31.4
0U157	275	15.7	30.7	19.3	16.1	1.17	1686	826	22.2
0U159	275	15.5	30.0	18.6	30.7	2.14	1686	815	40.9
0U183	275	15.2	31.0	18.7	23.3	1.85	1671	849	35.6
0U231	275	15.2	30.7	19.2	23.3	2.41	1671	819	42.9
0U232	275	15.2	31.0	19.2	14.3	1.48	1671	834	25.8
0U350	275	15.5	30.7	18.9	10.7	1.66	1671	814	28.6
0U460	275	15.2	30.5	18.9	10.0	2.03	1671	789	31.4
0U087	275	15.2	30.5	18.5	25.5	0.96	1568	833	19.3
0U252	275	15.2	30.5	18.8	21.0	2.31	1568	821	37.8
0U033	275	15.2	30.7	19.9	36.8	0.55	1671	842	15.5
0U034	275	15.2	30.0	19.9	37.0	0.59	1686	817	14.5
0U038	275	15.2	30.7	17.8	49.7	0.78	1686	815	17.8
0U056	275	15.2	30.5	18.2	41.1	0.97	1686	860	22.1
0U076	275	15.2	31.0	18.8	32.9	1.11	1671	841	23.1
0U082	275	15.2	30.7	21.4	27.8	1.13	1671	880	28.3
0U149	275	15.5	30.7	20.9	30.1	2.22	1672	833	50.6
0U193	275	15.7	30.7	21.2	26.6	2.59	1671	820	51.5
0U238	275	15.7	30.5	19.5	13.9	1.55	1686	824	26.8
0U244	275	15.2	30.7	19.4	16.8	1.85	1671	822	29.1
0U288	275	15.5	30.7	18.9	17.7	2 34	1686	765	36.0
0U354	275	15.2	30.7	19.3	12.8	2.05	1671	806	30.0

Table 8 Testdata from ref. [62.1].

3.3. Effectiveness factor for bending ν_b

The experimental investigation has shown that the effectiveness factor for bending ν_b mainly is a function of the average effective prestress in the section, defined by $A_p f_{se}/(bd_p f_c)$, and the span/depth ratio, defined by $1/d_p$.

Parametric analysis of available test results of unbonded prestressed tendon beams has indicated that the effectiveness factor ν_b is increasing almost linearly with the average effective prestressing of the section up to about 0.5 (see Fig. 5.). This phenomenon is probably due to the fact that the prestressing effectively delays the formation of cracks and the required stress redistribution, as assumed in the plastic theory, is strongly decreasing when increasing value of the average effective prestressing on the section.

It has also been found that ν_b dependence on the average effective prestress in the section $A_p f_{se}/(bd_p f_c)$ is a slightly decreasing function when increasing the span/depth ratio up to about 20 (see also figure 5.). This is probably due to the dowel action and to the fact that the required stress redistribution is increasing for increasing span/depth ratio in this interval.

From the parametric analysis of 99 test results of unbonded prestressed beams failed in flexure, the ν_b–value can be calculated approximately by the simple empirical formula

$$\nu_b = 0.2\left[1 + (20 - 0.6\, 1/d_p) A_p f_{se}/(bd_p f_c)\right] \qquad \left[\begin{array}{l} 1/d_p \ngtr 20 \\ A_p f_{se}/(bd_p f_c) \ngtr 0.5 \end{array}\right] \tag{2}$$

For simplicity in most practical cases the ν_b value can be taken as

$$\nu_b = 0.2\left[1 + 10\, A_p f_{se}/(bd_p f_c)\right] \qquad \left[A_p f_{se}/(bd_p f_c) \ngtr 0.5\right] \tag{3}$$

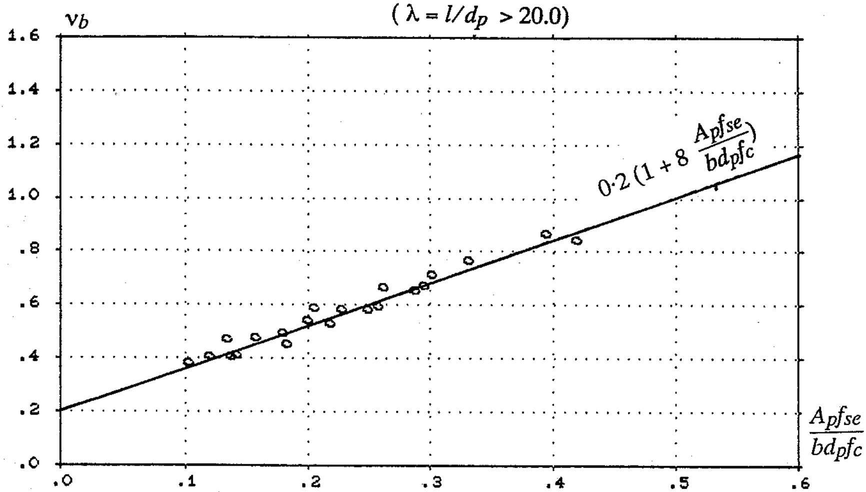


Figure 5 The ν_b dependence on $A_p f_{se}/(b d_p f_c)$. [69.3].

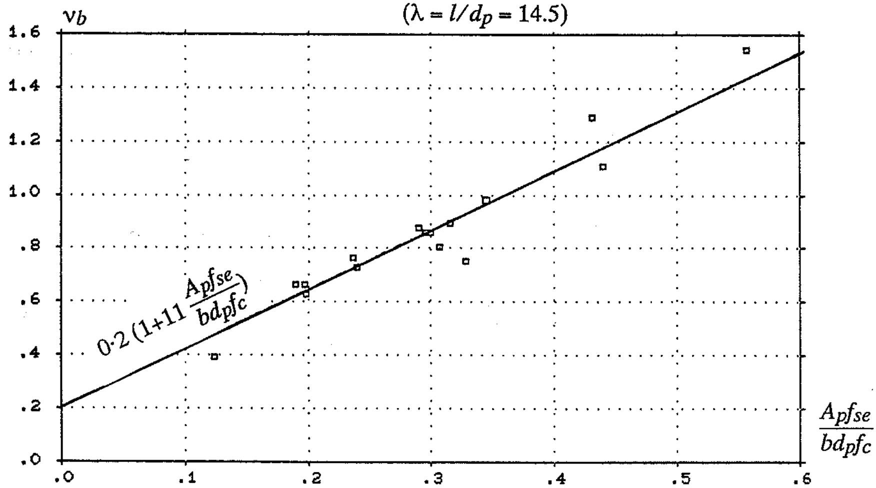


Figure 5 (continued). [62.1].

3.4 Comparison of tests with theory

The statistical values of the ratios of test values to theoretical values, which are found by equations (1) and (2), or (1) and (3), and the comparison of the test results with theoretical calculations are depicted in figure 6 and figure 7.

To demonstrate the general applicability of the plastic membrane theory to unbonded prestressed concrete beams, figure 8 and 9 show the close fit of the test results to the theoretical curves with the ν_b value calculated by (2) or (3).

It seems that the agreement between test and theory is quite satisfactory.

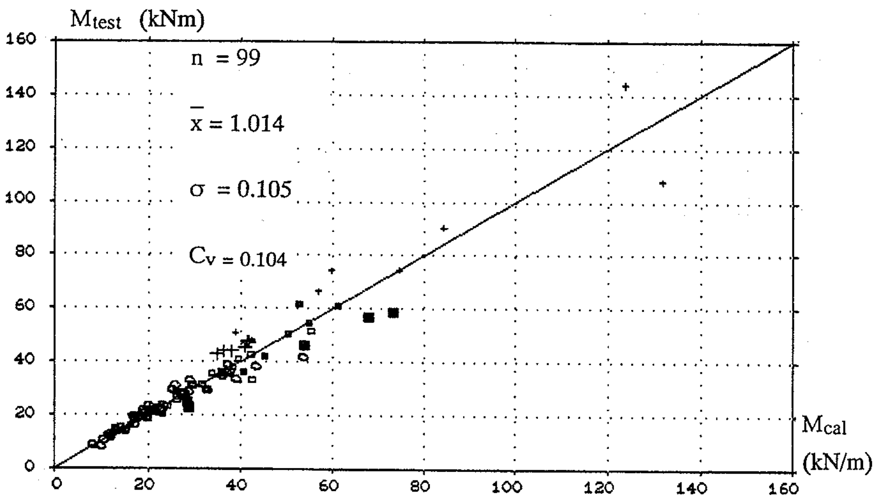


Figure 6 The comparison between tests and theoretical calculations, formulas (1) and (2).

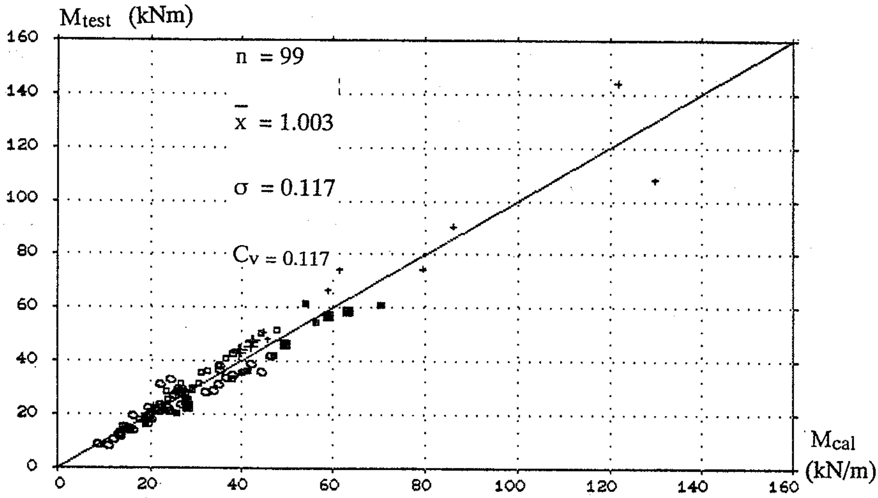


Figure 7 The comparison of tests with calculations, equations (1) and (3).

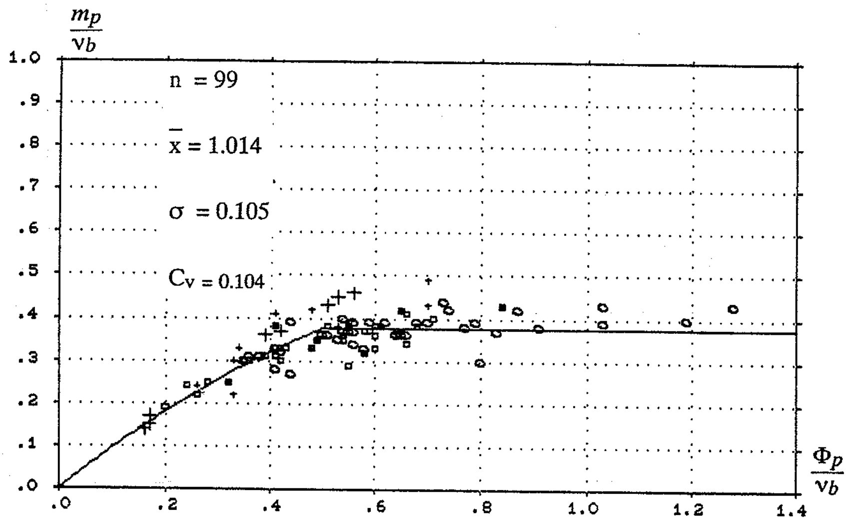


Figure 8 The tests compared with the theoretical curve. (ν_b calculated by (2)).

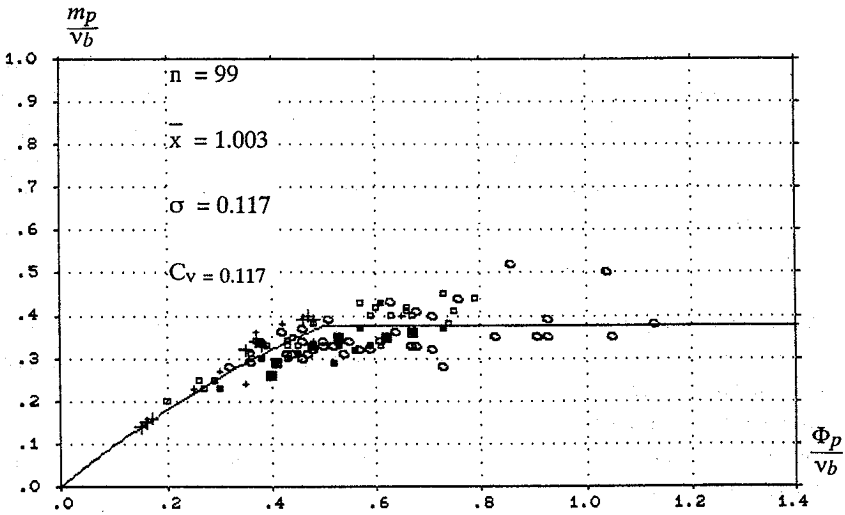


Figure 9 The tests compared with the membrane theory (ν_b found by equation (3)).

3.5 Conclusion

It may be concluded that the plastic theory for membrane action agrees very well for beams with unbonded tendons in bending.

4. Shear Strength of Prestressed Concrete Beams with Unbonded Tendons

How the missing bond of the prestressing tendons affects the shear carrying capacity has been investigated for beams without shear reinforcement [64.10], [72.3] and for beams with vertical stirrups [89.1]. Since it has been shown that the load carrying capacity of prestressed beams with bonded longitudinal reinforcement, whether it is web reinforced or not, can be solved by the plastic theory [88.10], it is natural to investigate,whether the shear carrying problem of prestressed beams with unbonded tendons can also be solved by this theory.

According to the plastic theory, prestress (bonded or unbonded) does not influence the theoretical shear solution in any way, because prestress introduces an internal, self–equilibrated stress system, which does not affect neither the failure mechanism, nor the stress distribution at collapse.
However, in reality the existence of an initial, non–zero compression stress state does influence the amount of stress redistribution which must take place before failure, and this has an effect on the ultimate load.

The aim of this section is to investigate from the existing test result, which conclusions may be drawn for using the plastic theory in prestressed beams with unbonded tendons.

4.1 Theoretical background

Under the assumptions mentioned before, the complete shear solutions of a simply supported stringer beam with vertical stirrups and subjected to concentrated loads were presented by Nielsen and Bræstrup [78.10], [84.1]. For conventional slender beams, in which the tensile reinforcement does not yield at failure, the shear strength can be stated in simple non–dimensional form as :

$$\frac{\tau}{f_c^*} = \begin{cases} \sqrt{\phi_v^* \, (1 - \phi_v^*)} & \phi_v^* \le \frac{1}{2} \\ \frac{1}{2} & \phi_v^* > \frac{1}{2} \end{cases} \tag{4}$$

Here we have introduced the average shear stress τ and the effective shear reinforcement degree ϕ_v^* as

$$\tau = \frac{V}{bh^*} \tag{5}$$

and

$$\phi_v^* = r_w \cdot \frac{f_{yw}}{f_c^*} \tag{6}$$

respectively. The parameter h* is the effective shear depth measured as the distance between the compressive and tensile stringers. For beams with T or I sections, it is natural to consider the distance from the centroid of the tensile reinforcement to the middle of the compressive flange as the effective shear depth h*.
Parametric analysis of a large number of available test results has indicated that the effectiveness factor ν mainly is a function of the uniaxial compressive strength f_c and the average effective prestress for bonded prestressed beams with shear reinforcement [88.10].

The ν –value slightly decreases with the increasing uniaxial compressive strength of the concrete since increasing concrete strength leads to decreasing ductility. The experimental investigation has also shown, that the ν–value is increasing almost linearly with the average effective prestressing of the section up to about 0.5 f_c This phenomenon is probably due to the fact that the prestressing effectively delays the formation of cracks and prevents the propagation of unstable inclined shear cracks. Therefore, prestressing increases the depth of the shear compressive zone of concrete and raises the shear capacity of the beams.

Besides the above two main factors, for smaller value of web reinforcement degrees the effect of the rather heavy flanges seem to raise the ν–value quite a bit.

From a practical point of view, the empirical ν formula has been suggested to be

$$\nu = \left[0.8 - \frac{f_c}{200}\right] \left[1 + 2\frac{F_{se}}{A_c f_c}\right] \qquad \left[\frac{F_{se}}{A_s f_c} \ngtr 0.5\right] \tag{7}$$

For prestressed concrete beams with bonded tendons, the web crushing criterion (4) and the empirical equation (7) give quite acceptable agreement with available test results, see [88.10].

4.2 Test results of prestressed beams with unbonded tendons failed in shear

In order to investigate the shear carrying behavior of beams with unbonded tendons, three test series have been carried out by Kordina et al. [84.2], [84.3], [87.1].

The shape of the cross section, the beam dimensions and the amount of reinforcement as well as the most important material properties are shown in figure 10 and table 9.

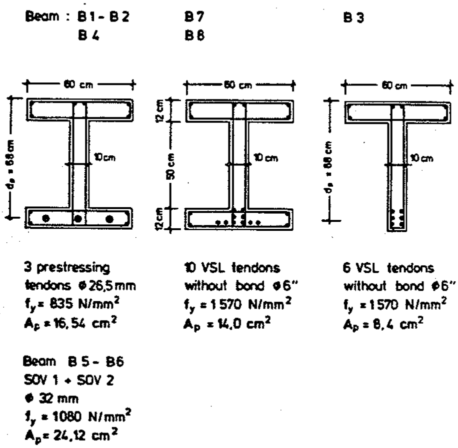


Figure 10 Cross sections and prestressing reinforcement of the test beams.

TEST DATA OF SHEAR REINFORCED BEAMS WITH UNBONDED TENDONS

b cm	h cm	d cm	h* cm
10.0	74.0	68.0	62.0

BEAM MARK	A_c cm^2	A_p cm^2	f_p MPa	A_s cm^2	f_y MPa	r_w %	f_{yw} MPa	f_c MPa	V_u kN	F_{se} kN	F_{su} kN	FM*
B-1	1940	16.54	835	6.28	428	0.18	446	18.0	225	380	596	TS
B-2	1940	16.54	835	2.26	446	1.34	431	24.0	400	700	1112	WC
B-3	1340	8.40	1570	0.28	446	1.34	431	27.0	338	784	994	WC**
B4-1	1940	16.54	835	1.13	475	0.19	475	48.0	375	948	1310	TS
B4-2	1940	16.54	835	1.13	475	0.54	449	48.0	483	1159	1435	TS
B4-3	1940	16.54	835	1.13	475	0.38	475	48.0	425	1070	1347	TS
B5-1	1940	24.12	1080	3.02	449	0.54	485	46.0	550	1389	1743	TS
B5-2	1940	24.12	1080	3.02	449	1.08	449	46.0	650	1330	1771	TS
B6-1	1940	24.12	1080	1.13	484	0.79	510	37.0	550	1349	1730	TS
B6-2	1940	24.12	1080	1.13	484	1.20	499	37.0	650	1460	1788	TS
B6-3	1940	24.12	1080	1.13	484	0.41	499	37.0	475	1276	1400	TS
B7-1	1940	14.00	1570	1.13	468	0.40	468	31.0	437	1030	1352	TS
B7-3	1940	14.00	1570	1.13	468	0.20	468	31.0	325	941	1003	TS
B8-1	1940	14.00	1570	1.13	469	0.78	498	35.0	575	1395	1790	TS
B8-3	1940	14.00	1570	1.13	469	0.40	508	35.0	437	1310	1372	TS
SOV1	1940	24.12	1080	-	-	2.09	468	39.0	788	1473	1883	WC
SOV2	1940	24.12	1080	-	-	1.34	474	37.0	667	829	1100	WC
SOV3	1940	24.12	1080	-	-	2.09	435	37.0	661	1100	1579	WC

* Failure mode, where TS symbolizes a beam failing in shear and WC symbolizes a beam failing as a result of web crushing.

** It looks like a bearing failure from the figure of the failure pattern. This beam has been excluded in the comparison between tests and theory.

Table 9.

4.3 ν–value for shear reinforced beams with unbonded tendons

The parametric analysis of these test results has shown that the major parameters governing the ν–value for prestressed beams with unbonded tendons are the same as those for beams with bonded tendons, but the ν–value for beams with unbonded tendons is only about 75% of that for beams with bonded tendons. The explanation for such a reduction is probably, that the required stress redistribution is increasing due to the missing bond of the prestressing tendons.

Considering the favorable benefit from heavy flanges for lower degrees of web reinforcement, the ν–value is not reduced in the case $\phi_v \geq 0.1$ If we denote the ν – value for beams with bond as ν_o, which is found by equation (7), then the ν formula for beams without bond may be taken as

$$\left.\begin{array}{ll} \nu_o = \left[0.8 - \dfrac{f_c}{200}\right]\left[1 + 2\dfrac{F_{se}}{A_c f_c}\right] & \left[\dfrac{F_{se}}{A_c f_c} \not> 0.5\right] \\ \nu = \nu_o & \phi_v \leq 0.1 \\ \nu = 0.75\ \nu_o & \phi_v > 0.1 \end{array}\right\} \qquad (8)$$

Here f_c should be in MPa.

4.4 Comparison of tests with theory

The statistical values of the ratios of test to theoretical values, which is found by equations (4) and (8), and the comparison of test results with theoretical calculations are presented in Fig. 11.

To demonstrate the general applicability of the plastic web crushing criterion, Fig. 12 shows the close fit of test results with the theoretical curve.

From figure 11 and figure 12 it appears that the agreement between tests and theory is quite satisfactory.

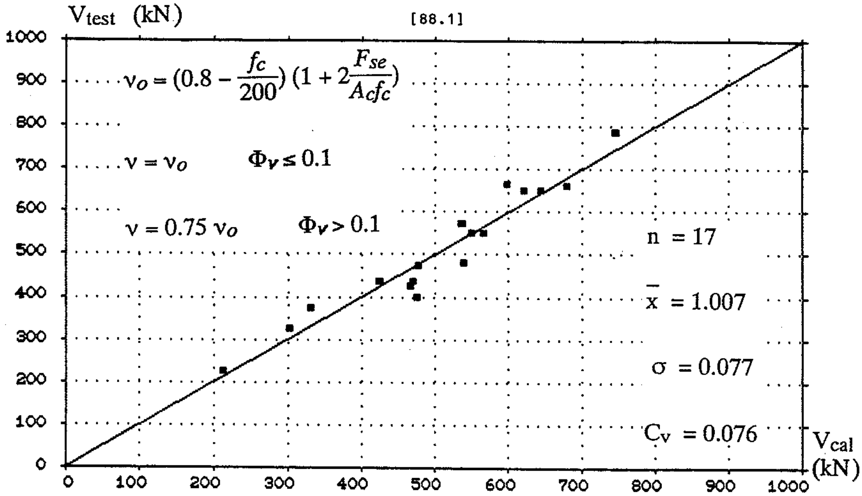


Figure 11 The comparison between tests and theory for prestressed beams with unbonded tendons.

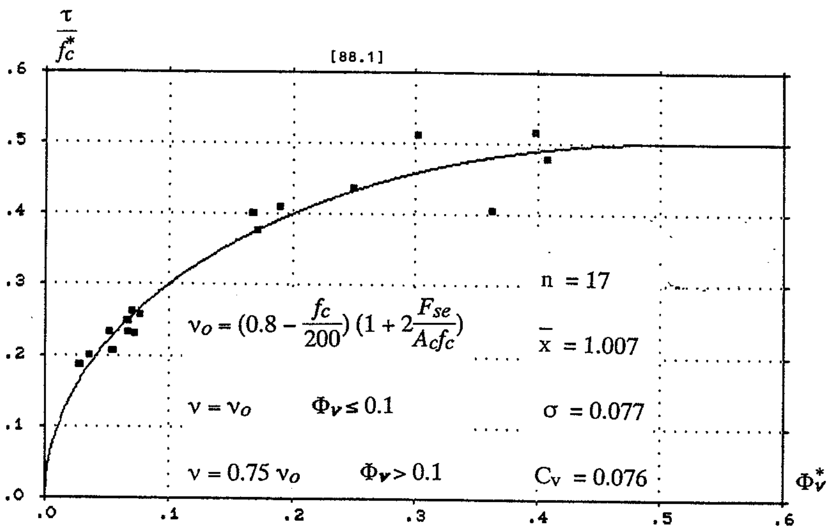


Figure 12 Theoretical shear capacity compared with test results.

4.5 Conclusion

The web crushing criterion is as suitable for beams with unbonded tendons as for beams with bonded tendons. The ν value for beams without bond is about $0.7 \sim 0.8$ of that for beams with bond. For smaller values of the web reinforcement degree, such a reduction is not necessary.

5. References

[56.2] **Janny, J.R. Hognestad, E. and McHenry, D.:** Ultimate Flexural Strength of Prestressed and Conventionally Reinforced Concrete Beams, Journal of the ACI, Feb. 1956, pp. 601–620.

[59.1] **Rüsch, H., Kordina, K. und Zelger, C.:** Bruchsicherheit bei Vorspannung ohne Verbund, Heft 130 der Schriftenreihe des DAfstb., Berlin, 1959.

[62.1] **Warwaruk, J., Sozen, M.A. and Siess, C.P.:** Strength and Behavior in Flexure of Prestressed Concrete Beams, Bulletin No. 464, Engineering Experiment Station, University of Illinois, Urbana, 1962.

[64.10] **Lorentsen, M.:** Shear and Bond in Prestressed Concrete Beams without Shear Reinforcement. Byggforsking Handlingar, nr. 47, 1964.

[69.3] **Pannell, F.N.:** The ultimate moment of resistance of unbonded prestressed concrete beams, Magazine of Concrete Research, Vol. 21, No. 66, March 1969, pp. 43–54.

[71.2] **Mattock, A. h., Yamazaki, J. and Kattula, B. T. :** Comparative Study of Prestressed Concrete Beams, With and Without Bond, Journal of the ACI, Feb. 1971, pp. 116–125.

[72.3] **Jena, B. and Pannell, F.N.:** The Diagonal Cracking Strength of Continuous Prestressed Concrete Beams. Magazine of Concrete Research, Vol. 24, No. 28, 1972.

[77.12] **Thomas, P.:** Les Cables non Adherents dans les Constructions en Beton, Resultats des Recherches Experimentales, C.S.T.C. – Rev. 1977, No. 4, Dezember 1977, pp. 2.18.

[78.5] **Ritz, P.:** Biegeverhalten von Platten mit Vorspannung ohne Verbund, Dissertation No. 6137, ETH, Zürich, Mai 1978.

[78.10] **Nielsen, M.P., Bræstrup, M.W., Jensen, B.C. and Bach, F.:** Concrete Plasticity: Beam shear – shear in joints – punching shear. Danish Society for Structural Science and Engineering/Structural Research Laboratory, Lyngby, 1978.

[80.1] **Trost, H., Cordes, H., Thormählen, U. und Hagen, H.:** Teilweise Vorspannung – Verbundfestigkeit von Spanngliedern und ihre Bedeutung für Risbildung und Risbreitenbeschränkung, Heft 310 der Schriftenreihe des DAfstb., Berlin, 1980.

[80.12] **Muller, J.:** Construction of Long Key Bridge. PCI Journal, Nov.–Dec. 1980, pp. 7–116.

[81.1] **Iványi, G. and Buschmeyer, W.:** Biegeversuche an Plattenstreifen mit Zentrischer Vorspannung ohne Verbund, Heft 16 der Forschungsberichte aus dem Fachbereich Bauwesen der Universität Essen, 1981.

[81.12] **Cooke, N. Park, R. and Yong, P.:** Flexural Strength of Prestressed Concrete Members with Unbonded Tendons, Journal of PCI, Nov.–Dec. 1981, pp. 52–80.

[83.1] **Iványi, G. and Buschmeyer, W.:** Versuche an Biegetragwerken mit Vorspannung ohne Verbund, Heft 20 der Forschungsberichte aus dem Fachbereich Bauwesen der Universität Essen, 1983.

[83.2] **Podolny, H. and Mireles, A.A.:** Kuwait's Bubiyan Bridge – a 3–d Precast Segmental Space Frame. PCI Journal, Jan.–Feb. 1983, pp. 68–1078.

[84.1] **Nielsen, M.P.:** Limit Analysis and Concrete Plasticity. Prentice–Hall, Inc., Englewood Cliffs, New Jersey, 1984.

[84.2] **Kordina, K., Hegger, J. und Teutsch, M.:** Anwendung der Vorspannung ohne Verbund. Schriftenreihe des Deutschen Ausschusses für Stahlbeton, H. 355, Berlin, Verlag W. Ernst & Sohn, 1984.

[84.3] Kordina, K., Teutsch, M. und Hegger, J.: Tray–und Verformungsverhalten von Spannbetonbauteilen in Segmentbauart mit Vorspannung ohne Verbund. Forschungsbericht des Instituts für Baustoffe, Massivbau und Brandschutz der TU Braunschweig, 1984.

[85.1] Hegger, J.: Einfluss der Verbundart auf die Grenztragfähigkeit von Spannbetonbalken, Heft 66, Braunschweig, 1985.

[87.1] Kordina, K. und Hegger, J.: Schubtragverhalten von Spannbetonbauteilen mit Vorspannung ohne Verbund. Schriftenreihe des Deutschen Ausschusses für Stahlbeton, H. 381, Berlin, Verlag W. Ernst & Sohn, 1987.

[88.1] Kordina, K. Hegger, J. and Teutsch, M.: Shear Strength of Prestressed Concrete Beams with unbonded Tendons, 1988.

[88.6] Nielsen, M.P., Andreasen, B.S. and Ganwei, C.: Dome effect in reinforced concrete slab stribs, Department of Structural Engineering, Technical University of Denmark. To be published.

[88.10] Chen Ganwei : Plastic Analysis of Shear in Beams, Deep Beams and Corbels. Ph.D. Thesis, Report Serie R No. 237, Department of Structural Engineering, Technical University of Denmark, Lyngby, 1988.

AFDELINGEN FOR BÆRENDE KONSTRUKTIONER
DANMARKS TEKNISKE HØJSKOLE

Department of Structural Engineering
Technical University of Denmark, DK–2800 Lyngby

SERIE R
(Tidligere: Rapporter)

R 230. RIBERHOLT, H.: Woodflanges under tension, 1988.
R 231. HOLKMANN OLSEN, N.: Implementation. 1988. (public. pending).
R 232. HOLKMANN OLSEN, N.: Uniaxial. 1988. (public pending)
R 233. HOLKMANN OLSEN, N.: Anchorage. 1988. (public pending)
R 234. HOLKMANN OLSEN, N.: Heat Induced. 1988. (public pending)
R 235. SCHEEL, HELLE: Rotationskapacitet. 1988. (public pending)
R 236. NIELSEN, MONA: Arbejdslinier. 1988. (public pending)
R 237. GANWEI, CHEN: Plastic Analysis of Shear in Beams. Deep Beams and Corbels. 1988.
R 238. ANDREASEN, BENT STEEN: Anchorage of Deformed Reinforcing bars. 1988.
R 239. ANDREASEN, BENT STEEN: Anchorage Tests with deformed Reinforcing Bars in more than one layer at a Beam Support. 1988.
R 240. GIMSING, N.J.: Cable–Stayed Bridges with Ultra Long Spans. 1988.
R 241. NIELSEN, LEIF OTTO: En Reissner–Mindlin Plade Element Familie. 1989.
R 242. KRENK, STEEN og THORUP, ERIK: Stochastic and Concrete Amplitude Fatigue Test of Plate Specimens with a Central Hole. 1989.
R 243. AARKROG, P., THORUP, E., KRENK, S., AGERSKOV, H. and BJØRN-BAK–HANSEN, J.: Apparatur til Udmattelsesforsøg. 1989.
R 244. DITLEVSEN, OVE and KRENK, STEEN: Research Workshop on Stochastic Mechanics, September 13–14, 1988.
R 245. ROBERTS, J.B.: Averaging Methods in Random Vibration. 1989.
R 246. Resumeoversigt 1988 – Summaries of Papers 1988. 1989.
R 247. GIMSING, N.J., JAMES D. LOCKWOOD, JAEHO SONG: Analysis of Erection Procedures for Cable–Stayed Bridges. 1989.
R 248. DITLEVSEN, O. og MADSEN, H.O.: Proposal for a Code for the Direct Use of Reliability Methods in Structural Design. 1989.
R 249. NIELSEN, LEIF OTTO: Simplex Elementet. 1989.
R 250. THOMSEN, BENTE DAHL: Undersøgelse af "shear lag" i det elasto–plastiske stadium. 1990.
R 251. FEDDERSEN, BENT: Jernbetonbjælkers bæreevne. 1990.
R 252. FEDDERSEN, BENT: Jernbetonbjælkers bæreevne, Appendix. 1990.
R 253. AARKROG, PETER: A Computer Program for Servo Controlled Fatigue Testing Documentation and User Guide. 1990.
R 254. HOLKMANN OLSEN, DAVID & NIELSEN, M.P.: Ny Teori til Bestemmelse af Revneafstande og Revnevidder i Betonkonstruktioner. 1990.
R 255. YAMADA, KENTARO & AGERSKOV, HENNING: Fatigue Life Prediction of Welded Joints Using Fracture Mechanics. 1990.
R 256. Resumeoversigt 1989 – Summaries of Papers 1989. 1990.
R 257. HOLKMANN OLSEN, DAVID, GANWEI, CHEN, NIELSEN, M.P.: Plastic Shear Solutions of Prestressed Hollow Core Concrete Slabs. 1990.
R 258. GANWEI, CHEN & NIELSEN, M.P.: Shear Strength of Beams of High Strength Concrete. 1990.
R 259. GANWEI, CHEN, NIELSEN, M.P. NIELSEN, JANOS, K.: Ultimate Load Carrying Capacity of Unbonded Prestressed Reinforced Concrete Beams. 1990.
R 260. GANWEI, CHEN, NIELSEN, M.P.: A Short Note on Plastic Shear Solutions of Reinforced Concrete Columbs. 1990.
R 261. GLUVER, HENRIK: One Step Markov Model for Extremes of Gaussian Processes. 1990.

Abonnement 1.7.1990 – 30.6.1991 kr. 130,–
Subscription rate 1.7.1990 – 30.6.1991 D.Kr. 130.–.

Hvis De ikke allerede modtager Afdelingens resumeoversigt ved udgivelsen, kan Afdelingen tilbyde at tilsende næste års resumeoversigt, når den udgives, dersom De udfylder og returnerer nedenstående kupon.

Returneres til
Afdelingen for Bærende Konstruktioner
Danmarks tekniske Højskole
Bygning 118
2800 Lyngby

Fremtidig tilsendelse af resumeoversigter udbedes af
(bedes udfyldt med blokbogstaver):

Stilling og navn: ....

Adresse: ....

Postnr. og -distrikt: ....

The Department has pleasure in offering to send you a next year's list of summaries, free of charge. If you do not already receive it upon publication, kindly complete and return the coupon below.

To be returned to:
Department of Structural Engineering
Technical University of Denmark
Building 118
DK-2800 Lyngby, Denmark.

The undersigned wishes to receive the Department's
List of Summaries:
(Please complete in block letters)

Title and name ....

Address...

Postal No. and district.................................

Country...