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A SHORT NOTE ON PLASTIC SHEAR SOLUTIONS OF REINFORCED CONCRETE COLUMNS

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Preface

This report is a part of an investigation on the influence of normal forces and prestressing on the shear capacity of reinforced concrete beams and columns.

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Resumé

I denne afhandling beskrives løsninger for forskydningsbæreevnen af bjælker og søjler af armeret beton påvirket til bøjning, forskydning og tryk. Løsningerne er baseret på plasticitetsteorien.

Summary

In this report solutions for the shear capacity of reinforced beams and columns subjected to bending, shear and compression are described. The solutions are based on the theory of plasticity.

Notation

a: Clear shear span

A_s: Cross–sectional area of axial reinforcement

A_{sw}: Cross–sectional area of shear reinforcement per unit length

b: Width of column section

C_1: Parameter, $$C_1 = \begin{cases} 1 - 2(n^* + \phi^*) & n^* < \frac{1}{2} - \phi^* \\ 0 & \frac{1}{2} - \phi^* \leq n^* \leq \frac{1}{2} + \phi^* \\ 1 - 2(n^* - \phi^*) & \frac{1}{2} + \phi^* < n^* \leq 1 + \phi^* \end{cases}$$

D_1: Parameter, $$D_1 = \begin{cases} \frac{\mu - 2(n^* + \phi^*)}{\lambda} & n^* < \frac{\mu}{2} - \phi^* \\ 0 & \frac{\mu}{2} - \phi^* \leq n^* \leq \frac{\mu}{2} + \phi^* \\ \frac{\mu - 2(n^* - \phi^*)}{\lambda} & \frac{\mu}{2} + \phi^* < n^* \end{cases}$$

f_c: Uniaxial compressive strength of concrete

f_c^*: Plastic compressive strength of concrete, defined as $f_c^* = \nu f_c$

f_t: Uniaxial tensile strength of concrete

f_t^*: Plastic tensile strength of concrete, defined as $f_t^* = \rho^* f_c^*$

f_y: Yield strength of axial reinforcement

f_{yw}: Yield strength of web reinforcement

h: Depth of cross–section of column

h^*: Effective shear depth

k: Material constant, $k = \frac{1 + \sin\varphi}{1 - \sin\varphi}$

N: Axial force

n^*: Effective axial force degree, defined as $n^* = N/bhf_c^*$

v: Relative displacement in yield line

V: Transversal loads. Shear force at cross section of column

α: Angle between yield line and relative displacement

β: Angle between yield line and column axis

η: Ratio, $\eta = \frac{\lambda}{\mu}$

ϵ_1, ϵ_2: Principal strains

θ: Angle

λ: Proportionality factor in flow law. Material constant, $\lambda = 1-(k-1)\rho^*$

μ: Coefficient of friction, $\mu = \tan\varphi$. Material constant, $\mu = 1-(k+1)\rho^*$

ν: Effectiveness factor for the compressive strength of concrete

ξ: Parameter, $$\xi = \begin{cases} n^* + \phi^* & n^* < \frac{k-1}{4k} - \phi^* \\ \frac{k-1}{4k} & \frac{k-1}{4k} - \phi^* \leq n^* \leq \frac{k-1}{4k} + \phi^* \\ n^* - \phi^* & \frac{k-1}{4k} + \phi^* < n^* \leq 1 + \phi^* \end{cases}$$

ρ^*: Effectiveness factor, $\rho^* = f_t^*/f_c^*$

σ_1, σ_2: Principal stresses

τ : Shear stress

φ : Angle of friction for concrete

ϕ^*: Effective reinforcement degree, defined as $\phi^* = A_s f_y / bh f_c^*$

ψ^* : Effective shear reinforcement degree, $\psi^* = A_{sw} f_{yw} / b f_c^*$

ψ_o^*: Critical shear reinforcement degree, $\psi_o^* = \dfrac{\sqrt{(\frac{a}{h^*})^2 + 4\omega(1-\omega)} - \frac{a}{h^*}}{2\sqrt{(\frac{a}{h^*})^2 + 4\omega(1-\omega)}}$

χ : Parameter, $\chi = \begin{cases} n^* + \phi^* & \frac{a}{h} \leq \tan\varphi \\ n^* - \phi^* & \frac{a}{h} > \tan\varphi \end{cases}$

ω : Parameter, $\omega = \begin{cases} n^* + \phi^* & n^* < \frac{1}{2} - \phi^* \\ \frac{1}{2} & \frac{1}{2} - \phi^* \leq n^* \leq \frac{1}{2} + \phi^* \\ n^* - \phi^* & \frac{1}{2} + \phi^* < n^* \leq 1 + \phi^* \end{cases}$

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I. Introduction

This paper deals with the shear capacity of short reinforced concrete columns with or without stirrups. These solutions are also valid for reinforced concrete beams subjected to transversal and axial loads (including compressive or tensile axial load). The corresponding bending capacity of such columns and beams can be found in [83.1] and [84.1]. The load carrying capacity is the lowest value found by a shear and a bending analysis.

II. Basic Assumptions

In the following sections the plastic shear solutions of reinforced concrete columns are treated based on the following assumptions:

1) The column is in a state of plane stress.

2) Any premature failures, such as anchorage failure and bearing or supporting failure are prevented by special anchorage provisions and adequate design of the bearings and load platens.

3) The reinforcement is rigid, perfectly plastic with a stress–strain relation for tension and compression as shown in Fig. 1. The yield strength of the reinforcement is denoted f_y. For steels without a definite yield point, f_y is defined in a suitable manner (e.g., as the 0.2% offset strength).

Furthermore, we assume that the reinforcing bars are only capable of carrying longitudinal tensile and compressive stresses, i.e., the dowel effects are neglected.

4) The concrete is rigid, perfectly plastic with the modified Coulomb's failure criterion as yield condition and the associated flow rule (normality condition) as constitutive equation. The yield condition and flow rule for concrete in plane stress are shown in Fig. 2 and Fig. 3.

Fig. 2 shows the yield condition for concrete with a non–zero tensile strength, while Fig. 3. with zero tension cut–off shows the so–called square yield locus for concrete in plane stress.

REINFORCEMENT

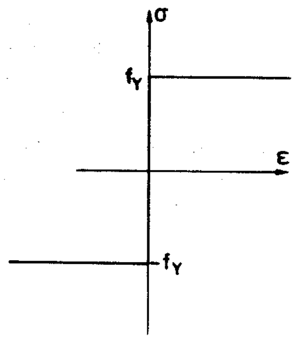


Figure 1. Uniaxial stress–strain relation for reinforcement.

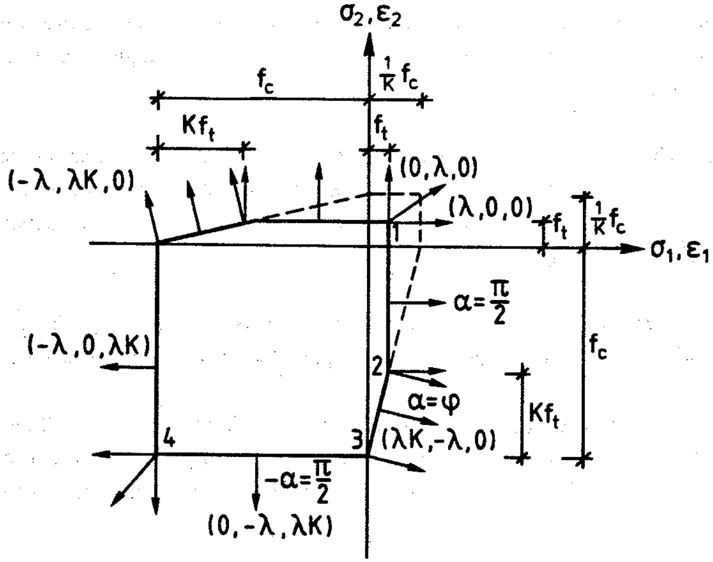


Figure 2. Yield condition and flow rule for a modified Coulomb material in plane stress.

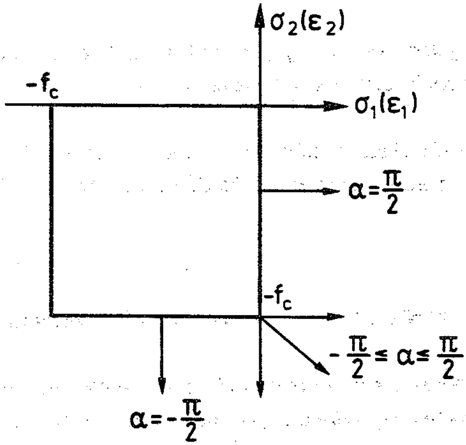


Figure 3. Square yield locus for concrete in plane stress.

The assumption that the materials are rigid and perfectly plastic means that any elastic deformations and work hardening effects are neglected and unlimited ductility is assumed.

Since concrete is not a perfectly plastic material, the plastic solutions have to be modified by replacing in the theoretical formulae the concrete strength by the effective plastic strengths, which are smaller than those strengths measured by standard tests.

If f_c is the uniaxial compressive strength of concrete measured by a standard compression test, then the plastic compressive strength of concrete f_c^* and the plastic tensile strength of concrete f_t^* are defined by

$$f_c^* = \nu f_c \tag{1}$$

$$f_t^* = \rho^* f_c^* = \rho^* \nu f_c \tag{2}$$

respectively. The quantities ν and ρ^* are called effectiveness factors. The value of ν and ρ^* normally must be determined by experiments.

5) For columns with shear reinforcement, the stirrup spacing is assumed to be sufficiently small to permit a continuous distribution of the equivalent stirrup forces.

III. Shear Capacity of Short Columns without Shear Reinforcement

In this section the shear carrying capacity of reinforced concrete columns without shear reinforcement is treated. Optimized upper bound solutions for rectangular columns are presented.

Fig. 4 shows part of a rectangular column subjected to an axial compressive force N and a transverse concentrated load V.

Consider a failure mechanism consisting of a single straight yield line inclined at the angle β to the column axis and running from A, which is one of the ends of the clear shear span, to point O, which may vary along the clear shear span. This yield line subdivides the beam into two rigid parts marked I and II in Fig. 4. Failure is assumed to take place along this yield line. The relative displacement rate is v, inclined at the angle α to the yield line. This failure mechanism is kinematically admissible.

Using upper bound technique, the complete solutions may be found in [88.1]. The best upper bound solutions and their valid regions are depicted in Fig. 5 without derivations. For a more detailed presentation, see [88.1]. In Fig. 5, the boundaries are plotted for $\rho^* = 0.06$ and $\phi^* = 0.15$.

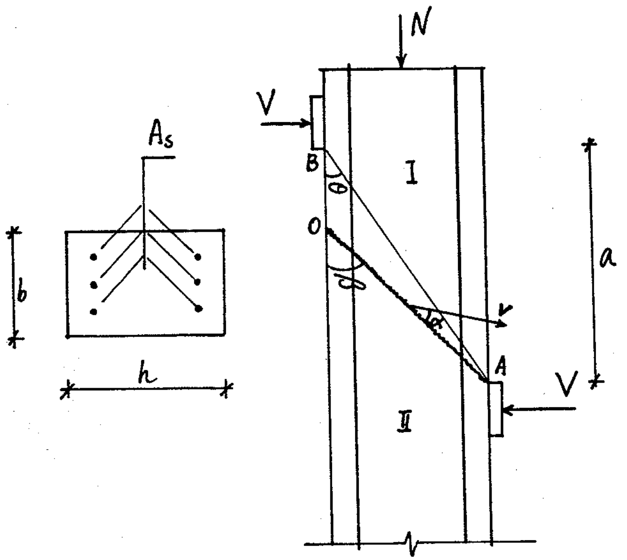


Figure 4. Failure mechanism in shear span for column without shear reinforcement.

The solutions are also shown in Fig. 6 and 7 for some values of the different parameters. In general, for low values of the normal force, the shear capacity increases with increasing normal force, while for high values of the normal force the shear capacity decreases with increasing normal force.

In Fig. 8 the influence of the different ρ^*– values to the shear capacity of columns has been shown for different $\frac{a}{h}$ and ϕ^* values. When we neglect the tensile strength of concrete ($\rho^* = 0$), i.e., if we take the square yield locus shown in Fig. 3 as the yield condition for concrete, the complete plastic solutions can be very much simplified into:

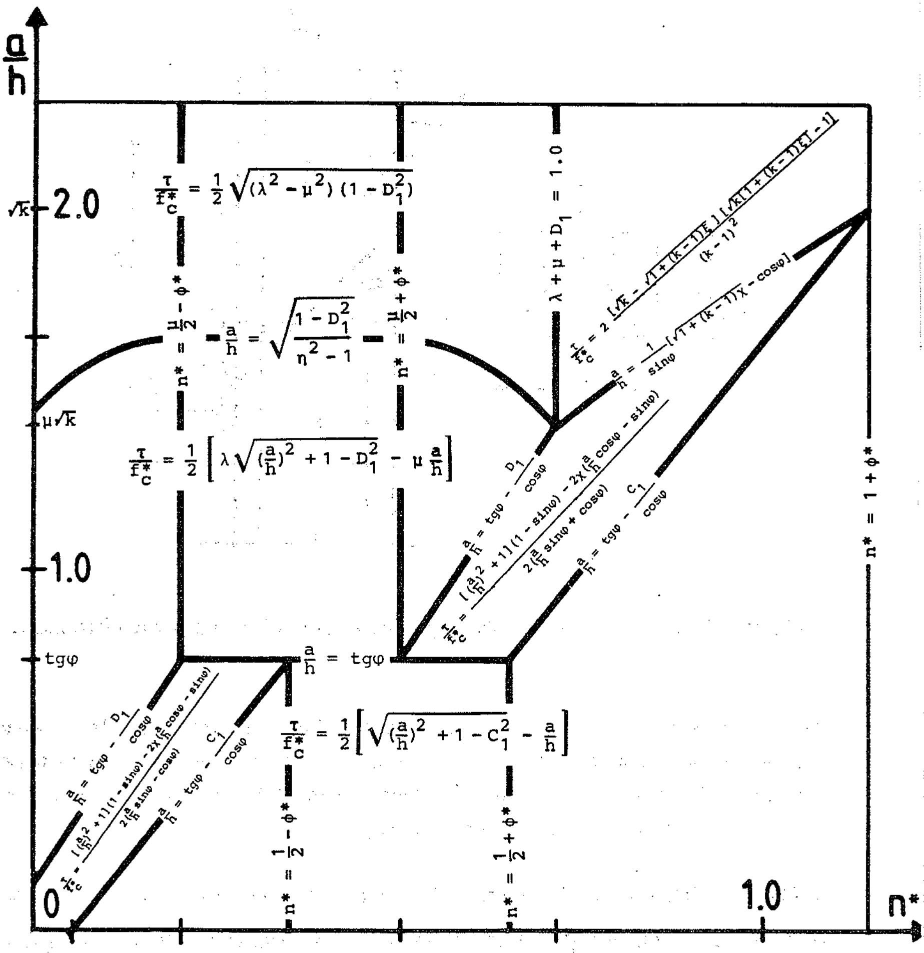


Figure 5. Regions of the plastic solutions for short columns subjected to axial and transversal loads.

$$\frac{\tau}{f_c^*} = \frac{1}{2}\left[\sqrt{\left(\frac{a}{h}\right)^2 + 1 - C_1^2} - \frac{a}{h}\right] \tag{3}$$

$$C_1 = \begin{cases} 1 - 2(n^* + \phi^*) \; ; & n^* + \phi^* < \frac{1}{2} \\ 0 \; ; & n^* - \phi^* \leq \frac{1}{2} \leq n^* + \phi^* \\ 1 - 2(n^* - \phi^*) \; ; & \frac{1}{2} < n^* - \phi^* \leq 1.0 \end{cases} \tag{4}$$

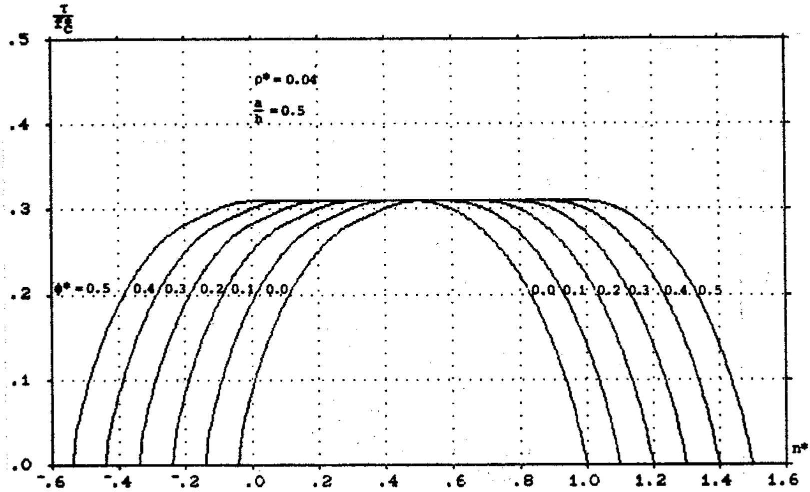


Figure 6. Shear carrying capacity of short columns subjected to transversal and axial loads versus the axial force degree for different longitudinal reinforcement degrees.

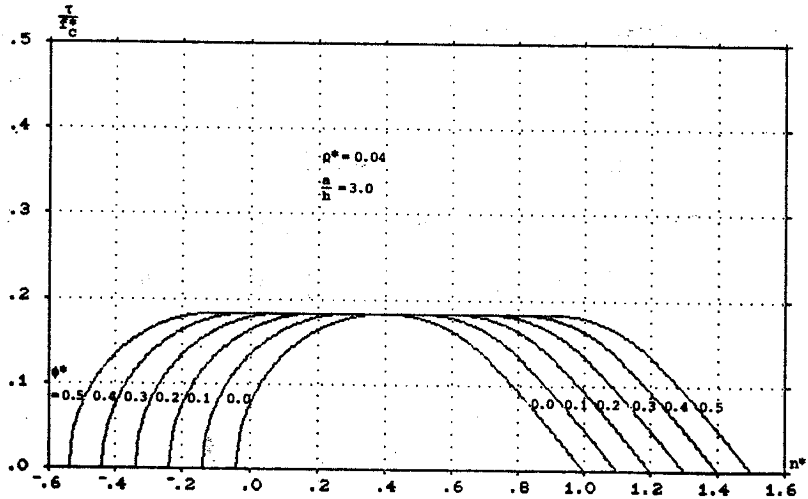


Figure 6. (Continued).

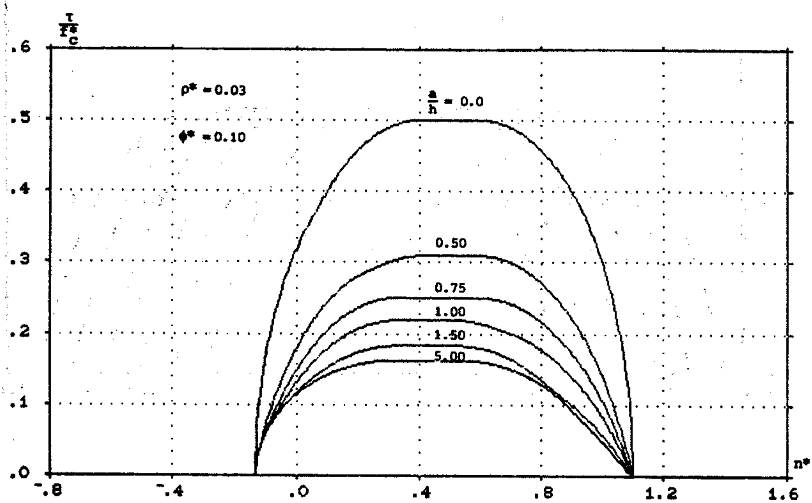


Figure 7. Shear carrying capacity versus the axial force degree for different shear span ratios.

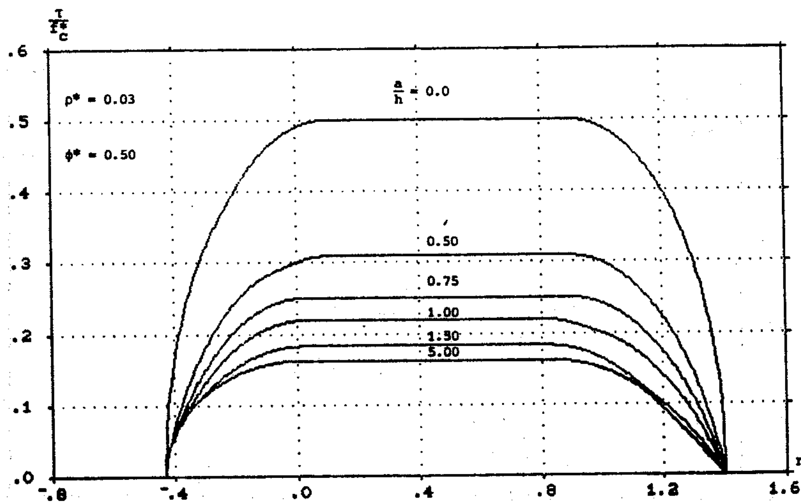


Figure 7. (Continued).

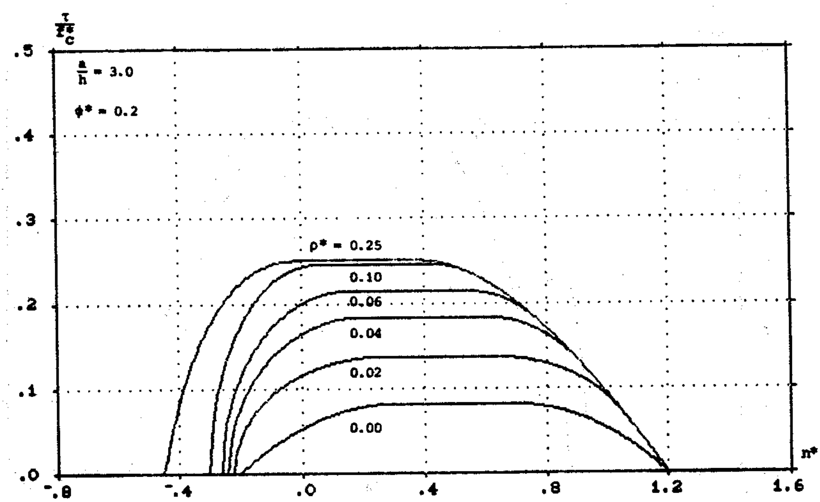


Figure 8. The influence of ρ^*– values on the shear capacity of short columns subjected to axial and transversal loads.

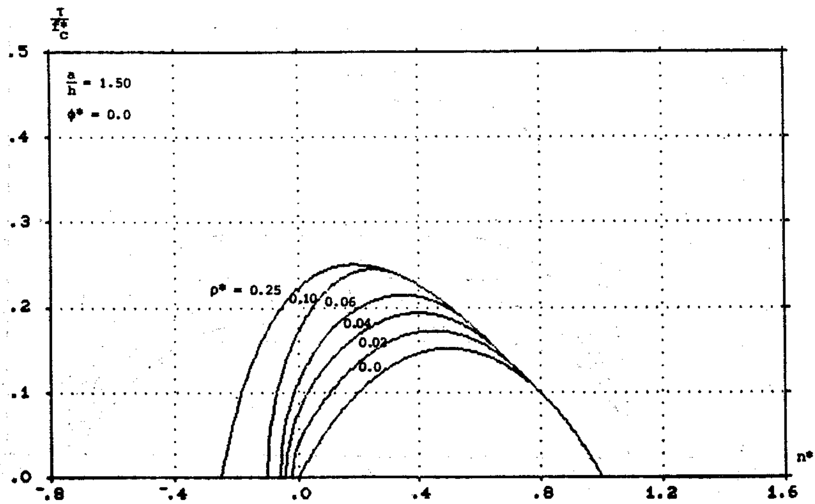


Figure 8. (Continued).

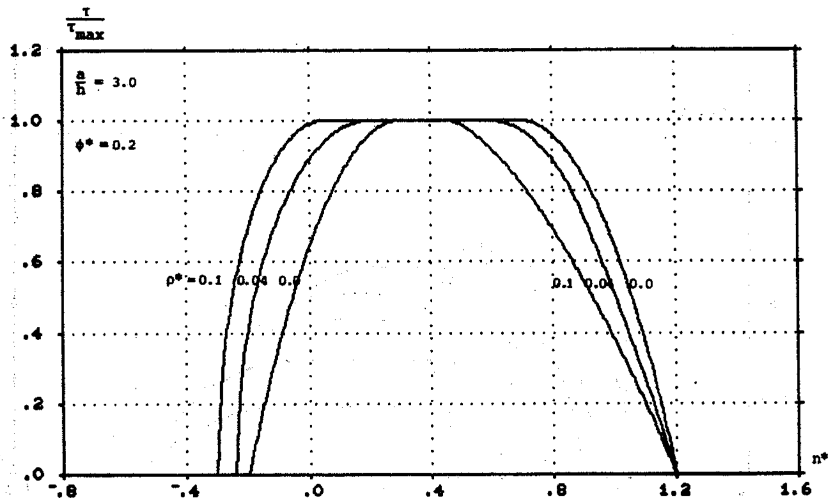


Figure 9. The influence of ρ^* on shear capacity for various $\frac{a}{h}$ and ϕ^*.

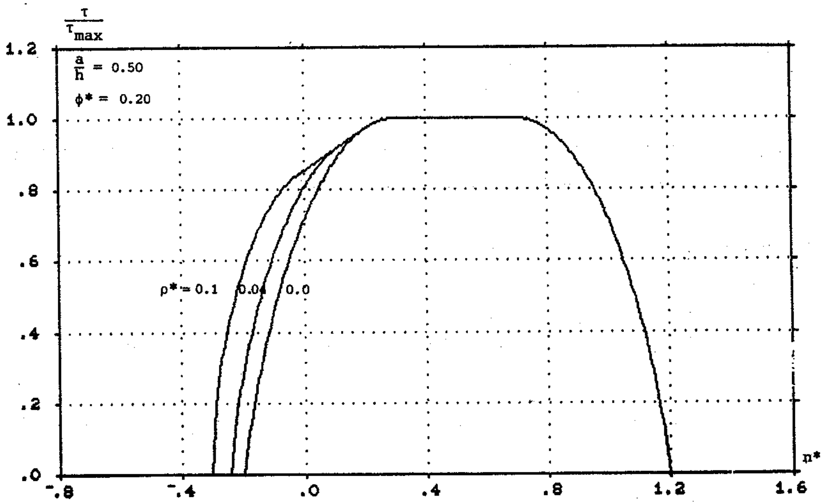


Figure 9. (Continued).

The simple solution (3) was found by M. Roikjær, see [77.1] and [78.1]. The difference between the case of considering the tensile strength of concrete or neglecting it can be seen in Fig. 9.

In the case of large axial compressive loads, the simple solution may overestimate the shear capacity for columns with large shear span ratio. For the small shear span ratio, the simple solution is the correct one.

IV. Shear Capacity of Short Columns with Shear Reinforcement

In this section, the lowest upper bound solutions will be given for stringer columns with stirrups.

Fig. 10 shows the shear span of a stringer column subjected to combined axial and transversal loads. The width and depth of the column are termed b and h*, respectively. The failure mechanism is assumed to consist of a single straight yield line inclined at angle β to the column axis. The relative displacement rate is v inclined at the angle α to the yield line. For columns with shear reinforcement, it is reasonable to neglect the tensile strength of concrete.

Using the upper bound method, the theoretical plastic shear solutions for short columns with stirrups have been found by the first author, Chen Ganwei.

The lowest upper bound solution are found to be

$$\frac{\tau}{f_c^*} = \begin{cases} \frac{1}{2}\left[\sqrt{\left(\frac{a}{h^*}\right)^2 + 4\omega(1-\omega)} - \frac{a}{h^*}\right] + \psi^* \cdot \frac{a}{h^*} & \text{for } \psi^* < \psi_o^* \\ 2\sqrt{\omega(1-\omega)\ \psi^*(1-\psi^*)} & \psi_o^* \le \psi^* \le \frac{1}{2} \\ \sqrt{\omega(1-\omega)} & \psi^* > \frac{1}{2} \end{cases} \qquad (5)$$

$$\omega = \begin{cases} n^* + \phi^* & n^* < \frac{1}{2} - \phi^* \\ \frac{1}{2} & \frac{1}{2} - \phi^* \le n^* \le \frac{1}{2} + \phi^* \\ n^* - \phi^* & \frac{1}{2} + \phi^* < n^* \le 1 + \phi^* \end{cases} \qquad (6)$$

The solutions (5) are also depicted in Fig. 11 showing the shear carrying capacity $\frac{\tau}{f_c^*}$ versus the effective axial force n^* for some values of main parameters, i.e., the effective shear reinforcement degree ψ^*, the effective longitudinal reinforcement degree ϕ^* and the effective shear span/depth ratio $\frac{a}{h^*}$.

As it can be seen very clearly from Fig. 11, in general, the shear capacity increases with increasing shear reinforcement degree ψ^* up to 0.5; for lower shear reinforcement degree, the shear capacity decreases with increasing shear span/depth ratio $\frac{a}{h^*}$. For relatively higher shear reinforcement degree, this influence of $\frac{a}{h^*}$ to shear capacity will disappear. For low values of the longitudinal reinforcement degree ϕ^* and axial force degree n^*, the shear capacity increases with increasing axial force degree n^*, while for high values of the axial force degree the shear capacity decreases with increasing axial force degree n*, no matter whether the longitudinal reinforcement degreeϕ^* is high or low.

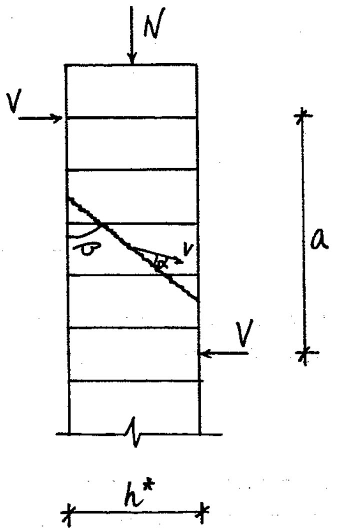


Figure 10. Failure mechanism of columns with stirrups and subjected to combined axial and transversal loads.

V. Experimental verification

Concerning the experimental verification, the reader is referred to [88.1]. In this work, empirical formulas for the effectiveness factors may also be found.

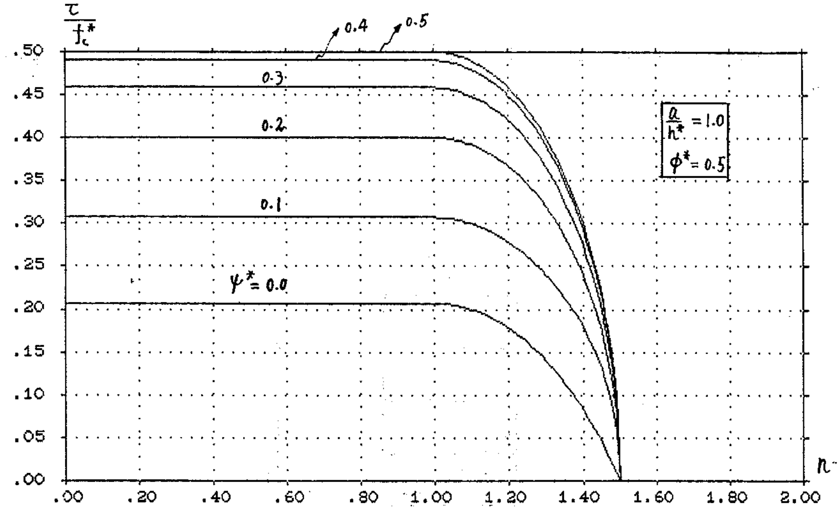
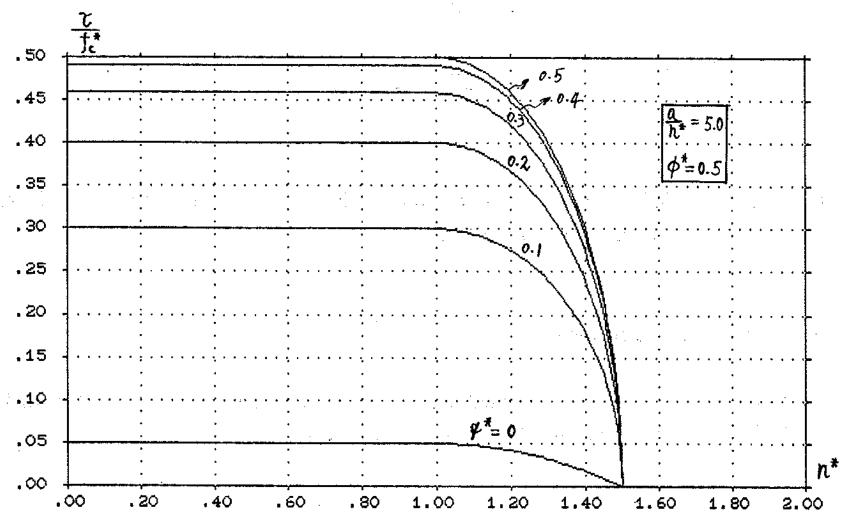


Figure 11. Shear carrying capacity versus the axial force degree with different shear reinforcement degrees.



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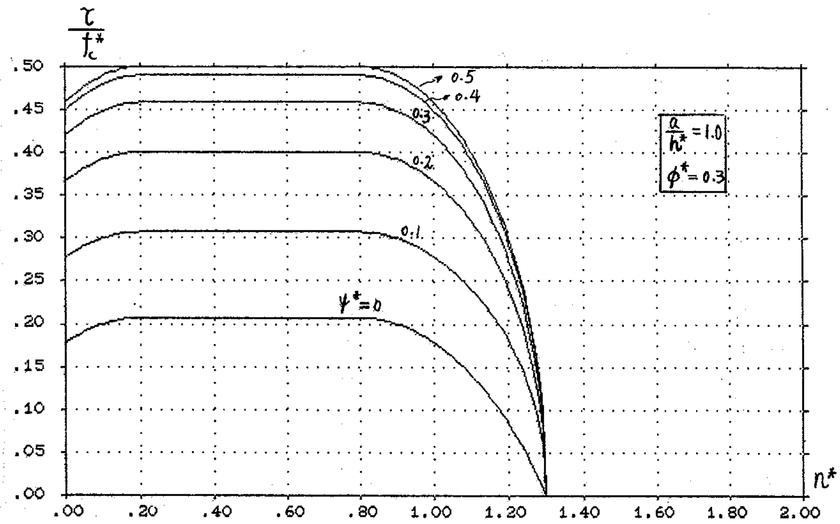

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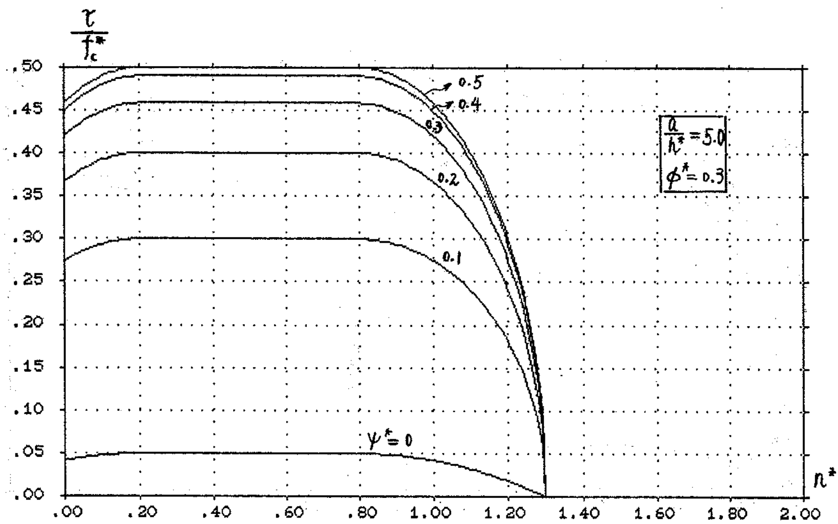

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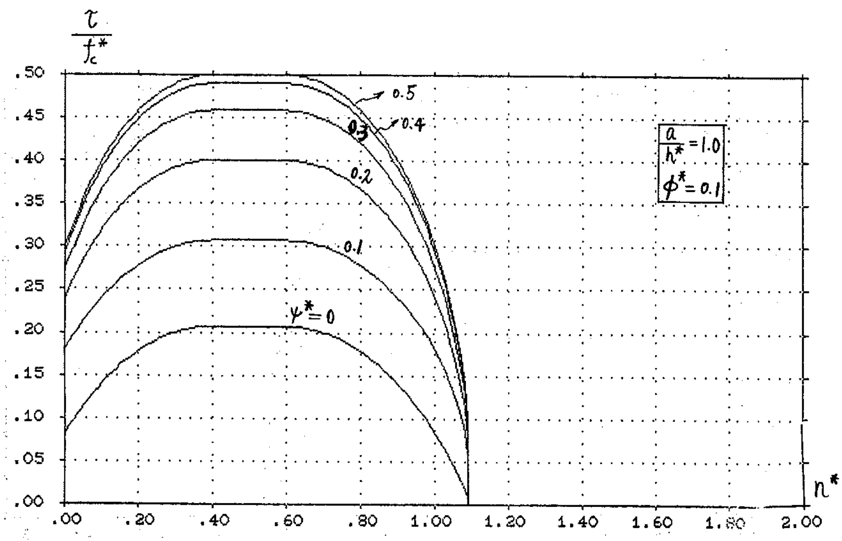

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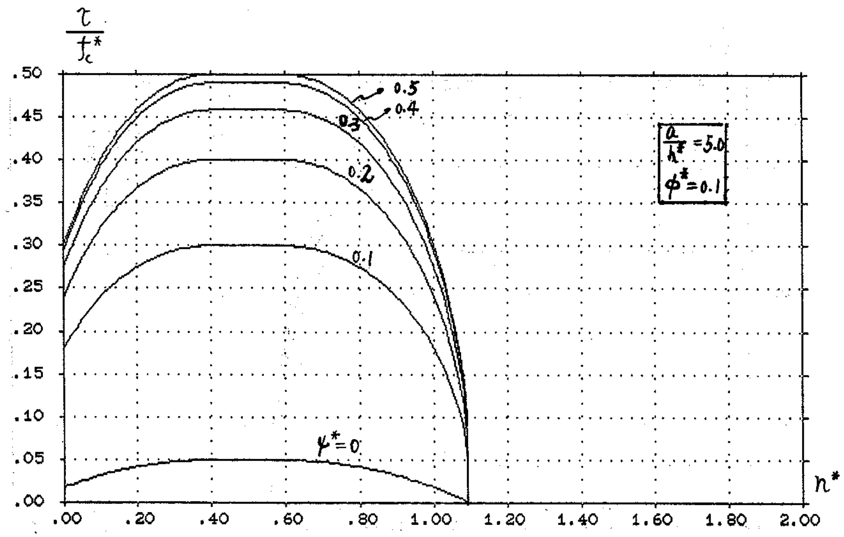

Figure 11. (Continued).

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